Term Structure of Interest Rates with Short-Run and Long-Run Risks*

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Abstract

To explain the violation of the expectations hypothesis, we propose a consumption-based model with recursive preferences, where long-run risks are separated from economic uncertainty and inflation is affected by the real side shocks. We show that the calibrated model matches well the nominal upward-sloping yield curve. Consistent with our model's implication, we find that variance risk premium based on interest rate swaptions is a strong predictor for short-horizon excess bond returns, whereas forward-rate based predictors drive long-horizon excess bond returns.

JEL Classification: G12, G13, G14

Keywords: Long-run risk, economic uncertainty, bond risk premia, variance risk premium, term structure of interest rates, expectations hypothesis

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Abstract

To explain the violation of the expectations hypothesis, we propose a consumption-based model with recursive preferences, where long-run risks are separated from economic uncertainty and inflation is affected by the real side shocks. We show that the calibrated model matches well the nominal upward-sloping yield curve. Consistent with our model's implication, we find that variance risk premium based on interest rate swaptions is a strong predictor for short-horizon excess bond returns, whereas forward-rate based predictors drive long-horizon excess bond returns.

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1 Introduction

The failure of the expectations hypothesis first documented by Fama and Bliss (1987) and Campbell and Shiller (1991) has attracted enormous attention in the asset pricing literature over the past decades. The fundamental challenge was (and still is) to find the underlying sources of bond return predictability. Uncovering these sources is important both for market participants and for monetary policy makers. Such sources can arguably be captured by a plethora of forecasting factors, such as a forward spread (Fama and Bliss, 1987), a forward rates factor (Cochrane and Piazzesi, 2005), realized jump risk measure (Wright and Zhou, 2009), a hidden term structure factor (Duffee, 2011), and macroeconomic variables (Ludvigson and Ng, 2009; Huang and Shi, 2012). However, what is the particular economic mechanism behind bond return predictability still remains an open question for the profession, and our paper focuses on this issue.

In this paper we build a long-run risk model with time-varying volatility of the volatility of the endowment process and money non-neutrality, with the purpose of understanding the economic drivers of the time variation of bond returns. The model allows us to disentangle short-run and long-run risks in bond returns. In particular, the endowment growth volatility of volatility (vol-of-vol) factor as in Bollerslev, Tauchen, and Zhou (2009) explains variation in short-horizon (one- and three-month) returns, whereas a persistent component as in Bansal and Yaron (2004) has more information for long-horizon (one-year) returns. While the existing literature since Fama and Bliss (1987) and Campbell and Shiller (1991) has documented predictability of long-horizon bond returns, papers exploring short-horizon predictability are almost non-existent, with the exceptions of Zhou (2009) and Mueller, Vedolin, and Zhou (2011). To our knowledge, this paper is the first to to reconcile these findings within a structural framework. Our proposed model includes both factors—endowment growth vol-of-vol factor and a persistent component of the aggregate growth in the economy—with Epstein-Zin-Weil recursive preferences and, therefore, allows for both types of predictability.

In order to match the nominal term structure of interest rates and bond return predictability, our model allows the inflation process to be affected by the endowment shock and a uncertainty channel and so our model features money non-neutrality. Bansal and Shaliastovich (2013) also link time variation in bond premia to a variation in volatility in real activity and inflation, but they do not model uncertainty process explicitly. The key in our model is the presence of real economic uncertainty, which enters through the time-varying volatility of volatility (vol-of-vol) of the endowment process. Since inflation in our model is affected by the real side growth and uncertainty shocks, prices in our model are implicitly affected by nominal uncertainty as well. The assumption that the real side shocks have effect on inflation results in the money non-neutrality feature—that is supported by previous theoretical and empirical studies, such as Pennacchi (1991) and Sun (1992).

There are three key results in our paper. The first result in our paper is that the time variation in the short-horizon bond risk premium is explained by the variance risk premium derived from the interest rate swaptions market. The sign of the variance risk premium is always positive, consistent with our structural model's prediction. In our model, variance risk premium is endogenously linked to the uncertainty factor—in fact, uncertainty factor is the only driver of the variance risk premium. Mueller, Vedolin, and Zhou (2011) also demonstrated short-horizon bond return predictability from equity variance risk premium, although sometimes marginally significant. In our case, variance risk premium derived from interest rate swaptions markets explains roughly 30 percent of the variation in one-month excess bond returns and roughly 20 percent in three-month Treasury excess returns.

The second result in our paper is that the variance risk premium has limited forecasting power for long-horizon returns, whereas factors like Fama-Bliss forward spread or Cochrane-Piazzessi factor are most important. This indicates that the latter two variables capture information related to the variation of the long-run risk factor—persistent component of the endowment growth—more than that of the short-run risk factor—uncertainty or volatility-of-volatility on the endowment growth. Theoretically, bond risk premium in our model is related to the variation of the persistent component and to the uncertainty factor, and we find that both factors are empirically important but along much different time horizons.

The third result comes from our calibration exercise and points out that the presence of the persistent component in the endowment growth helps fitting the upward slope of the Treasury yield curve. The absence of the long-run risk factor results in the flat or inverted yield curve, and so the presence of the long-run risk seems to be important for explaining the overall level of interest rates, in addition to its power for explaining the long-horizon bond return predictability.

Our results have important implications as to which factors are at work for explaining first and second moments of bond returns. It appears that the presence of the long-run risk factor helps explaining the level of the interest rates, while the vol-of-vol factor helps explaining the variation in the short-horizon interest rates. The variation in the long-horizon returns appears to be related to a different kind, possibly more longer-run volatility factor embedded in the aggregate endowment growth. While we do not model the two types of volatilities explicitly, there is a growing existing literature that argues for the existence of the short-run and long-run risk components of the aggregate volatility to study the variation of stock returns (Adrian and Rosenberg, 2008; Christoffersen, Jacobs, Ornthanalai, and Wang, 2008; Branger, Rodrigues, and Schlag, 2011; Zhou and Zhu, 2012, 2013). We are the first, to the best of our knowledge, to discover empirically that bond returns are also driven by a similar two-factor volatility structure.

The rest of the paper is organized as follows. Section 2 presents our long-run risk model with macro-economic uncertainty and asset pricing implications of the model, Section 3 discusses calibration of the Treasury yield curve implied by our model, Section 4 provides overview of the interest rate swaptions market and introduces swaptions-based variance risk premium measure, Section 5 describes all relevant data to our empirical exercise, and Section 6 presents empirical results. Finally, Section 7 concludes.

2 Model and Asset Pricing

2.1 Preferences

We consider a discrete-time endowment economy with recursive preferences for early resolution of uncertainty introduced by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1989):

$$U_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where δ is the time discount factor, $\gamma \geq 0$ is the risk aversion parameter, $\psi \geq 0$ is the intertemporal elasticity of substitution (IES), and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. Preference for early resolution of uncertainty is consistent with $\theta < 0$. Note that a special case of recursive preferences - constant relative risk aversion preferences - arises when $\gamma = \frac{1}{\psi}(\theta = 1)$.

Epstein and Zin (1989) show that the log-linearized form of the associated real stochastic discount factor m_t is given by:

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{c,t+1}, \tag{2}$$

where $g_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$ is the log growth of the aggregate consumption, $r_{c,t+1}$ is a log return on an aggregate wealth portfolio that delivers aggregate consumption as its dividend each time period. Note that the return on wealth is different from the observed return on the market portfolio because aggregate consumption is not equal to aggregate dividends. Consequently, the return on wealth is not observable in the data.

In order to solve for nominal prices in the economy, such as nominal bonds, we specify exogenous process for inflation π_{t+1}. The nominal discount factor $m_{t+1}^{\$}$ is equal to the real discount factor minus the inflation rate:

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}. \tag{3}$$

2.2 Economy dynamics

To solve for the equilibrium asset prices we specify consumption and inflation dynamics featuring time-varying expected consumption growth rates, stochastic volatility of the consumption growth rates, and time-varying vol-of-vol factor:

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \phi_e \sigma_{g,t} z_{x,t+1}, \\ g_{t+1} &= \mu_g + x_t + \sigma_{g,t} z_{g,t+1}, \\ \sigma^2_{g,t+1} &= a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1}, \\ q_{t+1} &= a_q + \rho_q q_t + \phi_q \sqrt{q_t} z_{q,t+1}. \end{aligned} \tag{4}$$

The vector of shocks follows *i.i.d.* normal distribution with mean zero and unit variance and shocks are assumed to be uncorrelated among themselves: $(z_{x,t+1},\, z_{g,t+1},\, z_{\sigma,t+1},\, z_{q,t+1}) \sim N(0, \mathcal{I})$. Relevant state variables in our model are (i) x_t - a predictable component of consumption growth, (ii) σ^2_t - stochastic volatility of consumption, and (iii) q_t - economic uncertainty variable.

The second pair of equations in (4) is new compared to the existing models of Bansal and Yaron (2004), Bollerslev, Tauchen, and Zhou (2009), and Bansal and Shaliastovich (2013). The economy features stochastic volatility of consumption growth rate, $\sigma_{g,t+1}$, which is affected by the vol-of-vol factor q_t. While q_t proxies uncertainty of the real side of the economy, it has a spill-over effect to the nominal side because inflation in the model is affected by the real side uncertainty shocks, as we show below. Thus, implicitly, economy is affected by both nominal and real side uncertainties. Inflation dynamics incorporates stochastic volatility and uncertainty factors that affect real economy specified in (4). We specify inflation π_{t+1} process as follows:

$$\pi_{t+1} = a_\pi + \rho_\pi \pi_t + \phi_\pi z_{\pi,t+1} + \phi_{\pi g} \sigma_{g,t} z_{g,t+1} + \phi_{\pi\sigma} \sqrt{q_t} z_{\sigma,t+1}, \tag{5}$$

where ρ_π is a persistence and $\frac{a_\pi}{1-\rho_\pi}$ is the long-run mean of the inflation process. There are three shocks that drive inflation process: (1) a constant volatility part ϕ_π with an autonomous shock $z_{\pi,t+1}$, (2) a stochastic volatility part $\phi_{\pi\sigma}\sigma_{g,t}$ that works through consumption growth channel $z_{g,t+1}$, and (3) another stochastic volatility part $\phi_{\pi\sigma}\sqrt{q_t}$ that works through the volatility channel $z_{\sigma,t+1}$. Note that the exposure to $z_{\pi,t+1}$ does not generate an inflation risk premium even if the volatility of this shock is time-varying, because this shock is exogenous. The last two terms in (5) generate inflation risk premium because real side shocks - stochastic volatility and uncertainty - affect inflation. Note also that since $\phi_{\pi g}$ and $\phi_{\pi\sigma}$ control inflation exposures to the growth and uncertainty risks, this process implicitly violates money-neutrality in the short run, but not in the long run.[1]

2.3 Pricing kernel

In equilibrium, the log wealth-consumption ratio z_t is affine in expected growth x_t, volatility of the growth σ_t^2, and economic uncertainty factor q_t:

$$z_t = A_0 + A_x x_t + A_\sigma \sigma_t^2 + A_q q_t. \tag{6}$$

Campbell and Shiller (1988) show that the return on this asset can be approximated as follows:

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{7}$$

where $\kappa_0 = \ln((1 + \exp \bar{z})) - \kappa_1 \bar{z}$, $\kappa_1 = \frac{\exp(\bar{z}))}{1+\exp(\bar{z})}$, and $\bar{z}$ is the average wealth-consumption ratio:

$$\bar{z} = A_0(\bar{z}) + A_\sigma(\bar{z})\bar{\sigma}^2 + A_q(\bar{z})\bar{q}. \tag{8}$$

[1]There is no violation of money neutrality in the long run because unconditional expectation of our inflation process is $\mathbb{E}\pi_t = \frac{a_\pi}{1-\rho_\pi}$.

The equilibrium loadings for (6) are derived in Appendix A.1:

$$\begin{aligned} A_x &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho_x}, \\ A_\sigma &= \frac{1}{2\theta(1-\kappa_1\rho_\sigma)}\left[\left(\theta-\frac{\theta}{\psi}\right)^2+(\theta\kappa_1A_x\phi_e)^2\right], \\ A_q &= \frac{1-\kappa_1\rho_q-\sqrt{(1-\kappa_1\rho_q)^2-\theta^2\kappa_1^4\phi_q^2A_\sigma^2}}{\theta(\kappa_1\phi_q)^2}. \end{aligned} \tag{9}$$

Recursive preferences along with the early resolution for uncertainty feature are crucial in determining the sign of the equilibrium loadings into the state variables. When intertemporal elasticity of substitution $\psi > 1$, the intertemporal substitution effect dominates the wealth effect. This means that agents invest more in a response to higher expected endowment growth, which contributes to a higher wealth-consumption ratio. Therefore, the loading on the expected consumption growth is positive, $A_x > 0$. In times of high volatility and/or uncertainty, agents sell off risky assets, and therefore, the wealth-consumption ratio falls. Thus, $A_\sigma < 0$ and $A_q < 0$.[2] At the same time, equation (6) underscores the difference between Bansal and Yaron (2004) and our model.

Euler equation imposes equilibrium restrictions on the asset prices:

$$\mathbb{E}[\exp(m_{t+1}+r_{t+1})] = 1, \tag{10}$$

This equation should hold for any asset, and for $r_{c,t+1}$ as well. The solutions of A coefficients in Eq. (6) are obtained using Euler equation (10), return equation (7), and conjectured z dynamics (6). Explicit form for the approximate solutions is given in the Appendix A.[3] This solution allows us to obtain a pricing kernel m_{t+1} as a function of state variables and shocks in the economy, and solve for equilibrium asset prices.

[2]The solution for A_q represents one of a pair of roots of a quadratic equation, but we pick the one presented in Eq. (9) as the more meaningful one. We elaborate on this choice in Section A.1.

[3]Bansal, Kiku, and Yaron (2012) check that their approximate solutions are very accurate when compared against numerical solutions, used, e.g., in Binsbergen, Brandt, and Koijen (2012).

Using the solution for consumption-wealth ratio, the analytical expression for the equilibrium stochastic discount factor can be also written as a linear combination of state variables and shocks in the economy. The innovation in the stochastic discount factor determines the sources and the compensations for risks in the economy:

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_g \sigma_{g,t} z_{g,t+1} - \lambda_x \sigma_{g,t} z_{x,t+1} - \lambda_\sigma \sqrt{q_t} z_{\sigma,t+1} - \lambda_q \sqrt{q_t} z_{q,t+1}, \tag{11}$$

where λ_g, λ_x, λ_σ, λ_q represent the market prices of risk of consumption growth, expected consumption growth, volatility, and uncertainty:

$$\begin{aligned} &\lambda_g = \gamma && \lambda_\sigma = (\theta - 1)\kappa_1 A_\sigma \phi_\sigma \\ &\lambda_x = (\theta - 1)\kappa_1 A_x \phi_e && \lambda_q = (\theta - 1)\kappa_1 A_q \phi_q \end{aligned} \tag{12}$$

The market price of the short-run consumption risk λ_g is equal to a coefficient of relative risk aversion γ. Other risk prices of risk crucially depend on our preference assumptions. If the agents have preference for early resolution of uncertainty ($\gamma > \frac{1}{\psi}$ or, equivalently, $\theta < 1$), then the market price of long-run risk $\lambda_x > 0$. In this case, positive shocks to consumption and expected consumption cause risk premia decrease, because in this case consumption-wealth ratio is expected to increase and investors will be buying risky assets. At the same time, when $\theta < 0$, market prices of risk of volatility and uncertainty are negative: Positive shocks to either volatility or uncertainty in the economy cause a sell-off of risk assets, thus, consumption-wealth ratio falls and risk premia increase. The quantities of risks in our economy are, of course, $\sigma_{g,t}$ and $\sqrt{q_t}$.

2.4 Asset prices

2.4.1 Risk-free rate

First, we price the real risk-free rate, which is the negative of the (log) price of the real one-period bond:

$$r_{f,t} = -p_t^1 = -\mathbb{E}_t[m_{t+1}] - \frac{1}{2}\mathrm{Var}_t[m_{t+1}], \tag{13}$$

The solutions for expectation and variance of the pricing kernel are given in Appendix A.2. Combining the appropriate terms, we state the solution for the real risk-free rate:

$$\begin{aligned}
r_{f,t} &= -\theta \ln \delta + \gamma(\mu_g + x_t) - (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\
&- (\theta - 1)[A_x(\kappa_1\rho_x - 1)x_t + A_\sigma(\kappa_1\rho_\sigma - 1)\sigma_{g,t}^2 + A_q(\kappa_1\rho_q - 1)q_t] \\
&- \frac{1}{2}\gamma^2\sigma_{g,t}^2 - \frac{1}{2}(\theta - 1)^2\kappa_1^2\left[A_x^2\phi_e^2\sigma_{g,t}^2 + (A_\sigma^2 + A_q^2\phi_q^2)q_t\right].
\end{aligned} \tag{14}$$

Note that the last two terms in (14) represent Jensen's inequality correction, while the terms in the middle line represent the time-varying risk-premia in real interest rates. The existence of this premia crucially depends on the assumption of the recursive utility, or $\theta \neq 1$. Moreover, the preference for early resolution of uncertainty ($\psi > 1$) insures that this risk premium is strictly positive. When $\theta = 1$, persistent component of endowment growth is absent and the volatility of consumption growth is constant, the model reduces to the case of CRRA utility and the risk-free rate reduces to a classical expression:

$$r_f = -\ln \delta + \gamma\mu_g - \frac{1}{2}\gamma^2\sigma_g^2. \tag{15}$$

The nominal risk-free rate is the negative of the (log) price of the nominal one period bond. Thus, it is equal to the real risk free rate plus the inflation compensation. The closed form for the nominal risk-free rate is derived in Appendix A.4:

$$
\begin{aligned}
r^{\$}_{f,t} &= -\theta \ln \delta + \gamma \mu_g + a_\pi - (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] - \frac{1}{2}\phi^2_\pi \\
&+ [\gamma - (\theta - 1)A_x(\kappa_1 \rho_x - 1)]\, x_t \\
&+ \left[-(\theta - 1)A_\sigma(\kappa_1\rho_\sigma - 1) - \frac{1}{2}\gamma^2 - \frac{1}{2}(\theta - 1)^2(\kappa_1 A_x \phi_e)^2 - \frac{1}{2}\phi^2_{\pi g} - \gamma\phi_{\pi g}\right] \sigma^2_{g,t} \qquad (16) \\
&+ \left[-(\theta - 1)A_q(\kappa_1\rho_q - 1) - \frac{1}{2}(\theta - 1)^2\kappa_1^2(A^2_\sigma + A^2_q\phi^2_q) - \frac{1}{2}\phi^2_{\pi\sigma} + (\theta - 1)\kappa_1 A_\sigma \phi_{\pi\sigma}\right] q_t \\
&+ \rho_\pi \pi_t.
\end{aligned}
$$

Since inflation is not an autonomous process, besides having a direct effect on the nominal rates, $\rho_\pi\pi_t$, it affects loadings on σ^2_t and q_t via additional terms, $\frac{1}{2}\phi^2_{\pi g}$ and $\frac{1}{2}\phi^2_{\pi\sigma}$, respectively. This results in money non-neutrality: inflation has an effect on the economy.

2.4.2 The $n-$period bond price

A general recursion for solving for the $n-$period nominal bond price is as follows:[4]

$$P^{\$,n}_t = \mathbb{E}_t\left[M^{\$}_{t+1} P^{\$,n-1}_{t+1}\right]. \qquad (17)$$

We assume that the (log) price of the $n-$period nominal bond $p^{\$,n}_t$ follows the affine representation of the real state variables x_t, σ^2_t, q_t and inflation π_t:

$$p^{\$,n}_t = B^{\$,n}_0 + B^{\$,n}_1 x_t + B^{\$,n}_2 \sigma^2_t + B^{\$,n}_3 q_t + B^{\$,n}_4 \pi_t. \qquad (18)$$

We solve for the nominal bond state loadings $B^{\$,n}_i$, $i = 0, \ldots, 4$ using the above recursion.[5] The nominal $n-$period nominal yield is defined as $y^{\$,n}_t = -\frac{1}{n}p^{\$,n}_t$. The log zero-period nominal bond price today $p^{\$,0}_t = 0$ (the log one-period bond price $y^{\$,1}_t = r^{\$}_{f,t}$). This gives us the initial conditions for the solution: $B^{\$,0}_i = 0$, $i = 0, \ldots, 4$, which, along with the state loadings, allow

[4]The solution for the $n-$period real bond price is provided in Appendix A.3.
[5]The solution is provided in Appendix A.5, equation (73).

to solve explicitly for the $n-$period nominal bond price.

2.4.3 Bond risk premium

Let $rx_{t+1}^{\$,n-1}$ be the bond excess return from t to $t+1$ for an $n-$period nominal bond holding one period. Then its expected value, or nominal bond risk premium, $\text{brp}_t^{\$,n}$, is given by the covariance between the nominal pricing kernel $m_{t+1}^{\$,n-1}$ and the nominal bond price $p_t^{\$,n}$:

$$
\begin{aligned}
\text{brp}_t^{\$,n} &= \text{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] \\
&= \left[-(\gamma+\phi_{\pi g})B_4^{\$,n-1}\phi_{\pi g} + (\theta-1)\kappa_1 A_x B_1^{\$,n-1}\phi_e^2\right]\sigma_{g,t}^2 \\
&+ \left[((\theta-1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}) + (\theta-1)\kappa_1 A_q B_3^{\$,n-1}\phi_q^2\right] q_t \\
&- B_4^{\$,n-1}\phi_\pi^2 \\
&\equiv \beta_1^{\$,n-1}\sigma_{g,t}^2 + \beta_2^{\$,n-1} q_t - B_4^{\$,n-1}\phi_\pi^2.
\end{aligned} \tag{19}
$$

The first two terms in (19) reflect consumption and uncertainty premiums amplified by the endogenous inflation shock parameters $\phi_{\pi g}$ and $\phi_{\pi\sigma}$ while the third term captures the autonomous inflation shock through ϕ_π. Note that the effect of the long-run risk captured by A_x amplifies the overall contribution of the consumption risk, $\sigma_{g,t}$. This effect is absent in Zhou (2011), Mueller, Vedolin, and Zhou (2011), and thus, makes it more difficult to explain the upward sloping term structure of the nominal yield curve.

2.4.4 Bond return predictability

Bollerslev, Tauchen, and Zhou (2009) show that the equity variance risk premium – the difference in expectations of the equity variance under risk-neutral and physical measures – is a useful predictor of time-variation in aggregate stock returns. Motivated by this result, we apply this measure to understand time variation in bond returns. While we do not derive the bond variance risk premium, it is fair to assume that temporal variation in stock and bond markets is correlated. In our model, time-varying variance risk premium arises endogenously:

$$\begin{aligned}\mathrm{VRP}_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\sigma^2_{r,t+1}\right] - \mathbb{E}_t^{\mathbb{P}}\left[\sigma^2_{r,t+1}\right] \\ &= (\theta-1)\kappa_1\left[A_\sigma(1+\kappa_1^2 A_x^2\phi_e^2) + A_q\kappa_1^2\phi_q^2(A_\sigma^2 + A_q^2\phi_q^2)\right] q_t.\end{aligned} \tag{20}$$

As equation (20) shows, time variation in the variance risk premium is due solely to time-variation in uncertainty state variable q_t. Note that consumption growth volatility does not affect the variance risk premium (and thus, bond return predictability). Also, recursive preferences $\gamma \neq \frac{1}{\psi}$ along with the early resolution of uncertainty ($\psi > 1$) deliver positive variance risk premia. So, the dual assumption of recursive preferences and presence of economic uncertainty is crucial for understanding bond return predictability. The common factor q_t in the nominal bond risk premium (19) and the variance risk premium (20) suggests that the latter should capture some time variation of the former. Thus, ignoring a measurement error, in a regression

$$\mathrm{brp}_t^{\$,n} = a + b\mathrm{VRP}_t, \tag{21}$$

the model-implied slope coefficient b and R^2 are given respectively,

$$b = \frac{\mathrm{Cov}(\mathrm{brp}_t^n, \mathrm{VRP}_t)}{Var(\mathrm{VRP}_t)} = \frac{\beta_2^{\$,n-1}}{(\theta-1)\kappa_1\left[A_\sigma(1+\kappa_1^2A_x^2\phi_e^2) + A_q\kappa_1^2\phi_q^2(A_\sigma^2 + A_q^2\phi_q^2)\right]} \tag{22}$$

and R2:

$$R^2 = \frac{b^2\mathrm{Var}(\mathrm{VRP}_t)}{\mathrm{Var}(\mathrm{brp}_t^{\$,n})} = \frac{\left(\beta_2^{\$,n-1}\right)^2\mathrm{Var}(q_t)}{\left(\beta_1^{\$,n-1}\right)^2\mathrm{Var}(\sigma^2_{g,t}) + \left(\beta_2^{\$,n-1}\right)^2\mathrm{Var}(q_t)}. \tag{23}$$

As discussed in Section 2.4.4, the constant volatility of volatility case implies no time variation in variance risk premium, and therefore $R^2 \equiv 0$. The other corner case, captured by the absence of the long-run risk component, implies $R^2 = 1$, which is the case that the variance risk premium can perfectly predict the bond risk premia, and the empirical predictability pattern cannot be replicated. Metrics (22)-(23) can be used to evaluate whether the proposed variable, and also proposed inflation dynamics can reproduce the empirical pattern of bond return predictability.

3 Calibration

In this section we discuss the calibration of the yield curve implied by the real side model (4) and inflation process (5). We consider two benchmark cases of parameters. One benchmark case follows Bansal and Yaron (2004, BY) and Bollerslev, Tauchen, and Zhou (2009, BTZ) that match the equity premium. In particular, we calibrate the yield curve with and without the long-run risk component. BTZ differs from BY in that it incorporates time-varying volatility of volatility in the model, and we differ from BTZ in that we add the long-run risk to the real side of the model and also model inflation necessary to derive implications for the nominal bond pricing.

We start with the calibration of the real economy, in particular, preferences, endowment and uncertainty processes. Panel A of Table 1 provides these calibration values. We follow BY choice for our preference choice parameters, by setting subjective time discount factor $\delta = 0.997$, $\gamma = 8$, and $\psi = 1.5$.[6] Volatility parameters a_σ and ρ_σ are set such that the unconditional expectation $\mathbb{E}\sigma_t^2 = 0.0078^2$, which is the value of the unconditional volatility process used by Bansal and Yaron (2004). Uncertainty parameters a_q, ρ_q, and ϕ_q are calibrated according to Bollerslev, Tauchen, and Zhou (2009) by setting the long-run level of uncertainty process $\mathbb{E}q_t = 10^{-6}$. Our choice of the volatility persistence parameter $\rho_\sigma = 0.978$ and the vol-of-vol persistence parameter $\rho_q = 0.8$ is broadly consistent with the estimates of Bollerslev, Xu, and Zhou (2013), who find that the long-run risk volatility (proxied by σ_{g_t}) is more persistent than the short-run risk (proxied by q_t). Thus, this choice of the parameters links our model calibration with the next empirical section where we show that these two types

[6] BY and BTZ use $\gamma = 10$, but in our model slightly lower value of γ works reasonably well.

of risks in the bond premia are disentangled.

Panel B provides the calibration parameters for inflation process. Inflation level a_π and persistence ρ_π are set such that the average annualized annualized inflation rate $\mathbb{E}_\pi = \frac{a_\pi}{1-\rho_\pi} = 2\%$.[7] Persistence of the inflation rate is set to $\rho_\pi = 0.95$ (which may be justified for data after 1980s and especially after 2008). Using these values for $\mathbb{E}_\pi$ and ρ_π, implied $a_\pi = 0.02/12 \times 0.05 = 8 \times 10^{-5}$. Variance parameters are set such that the total annualized inflation volatility is 2%. We assume that the first (autonomous) shock contributes one half to the total variance while other two shocks contribute equally to the remaining half of the total variance of inflation process.[8] The total unconditional variance of the inflation process is set as follows:

$$Var[\pi] = \frac{1}{1-\rho_\pi^2}\left[\phi_\pi^2 + \phi_{\pi g}^2 \frac{a_\sigma}{1-\rho_\sigma} + \phi_{\pi\sigma}^2 \frac{a_q}{1-\rho_q}\right]. \tag{24}$$

Since $\rho_\pi = 0.95$, $\frac{a_\sigma}{1-\rho_\sigma} \equiv \mathbb{E}\sigma^2 = 0.0078^2$, $\frac{a_q}{1-\rho_q} \equiv \mathbb{E}q = 10^{-6}$ the total unconditional inflation variance on a monthly basis is:

$$\left[\phi_\pi^2 + \phi_{\pi g}^2 \times 0.78^2 \times 10^{-4} + \phi_{\pi\sigma}^2 \times 10^{-6}\right] = 0.02^2/12 \times (1 - 0.95^2) = 3.25 \times 10^{-6}. \tag{25}$$

Thus, the contribution of the first shock to the total inflation variance is $0.5 \times 3.25 \times 10^{-6} = 1.625 \times 10^{-6}$, implying $\phi_\pi = 0.0013$. The contribution of the second and third term is equal to each other and equal to $0.25 \times 3.25 \times 10^{-6} = 8.125 \times 10^{-7}$. Thus, the implied $\phi_{\pi g} = (8.125 \times 10^{-7}/(0.78^2 \times 10^{-4}))^{1/2} = -0.0385$. As in BS, it is important for fitting upward-sloping yield curve that the correlation between inflation and endowment growth is negative, thus negative sign for this loading coefficient. Last, the implied $\phi_{\pi\sigma} = (8.125 \times 10^{-7}/10^{-6})^{1/2} = 28.5$.

Finally, Panel C of Table 1 provides Campbell-Shiller log-linearization constants κ_0 and κ_1.

[7] Our inflation rate is more consistent with the current Fed target and lower than the one in Bansal and Shaliastovich (2013, BS), who set it at 3.61% (see their Table 5).

[8] Equal distribution of variance among the shocks results in slight overshooting of the model-implied interest rates levels relative to those in the sample.

Figure 1 reports our calibration results. First, both panels show the average nominal yield curve (blue solid line) in January 1991 - December 2010 sample period. Second, both panels show the nominal yield curve (red dashed line) implied by our model without (Panel A) and with (Panel B) the long-run risk component x_t. It is obvious from Panel A that absent long-run risk the model is not successful in fitting the upward-sloping yield curve, even with the presence of economic uncertainty in the model. Alternatively, Panel B shows that improvement due to the slow-moving predictable component in the endowment growth is dramatic. Indeed, our model with x_t appears to successfully capture the slope and the level of the curve. The conclusion of this calibration exercise is that the level of the interest rates appears to be tightly linked to the slow-moving predictable component in the endowment growth.

4 Construction of the Variance Risk Premium

In this section we first overview the mechanics of interest rate swaptions and then discuss construction of swaption-implied variance risk premium based on the *variance contract on swap rates* defined in Li and Song (2014).

4.1 Interest rate swaptions

Consider a forward start fixed versus floating interest rate swap with a start date T_m and maturity date T_n. The fixed annuity payments are made on a pre-specified set of dates, $T_{m+1} < T_{m+2} < \cdots < T_n$, with the intervals equally spaced by δ, which equals six months in US swaption markets. The floating payments tied to the three-month LIBOR are made quarterly at $T_{m+1} - \delta/2$, T_{m+1}, $T_{m+2} - \delta/2$, T_{m+2}, $\cdots$, $T_n - \delta/2$, and T_n.[9]

At time T_m, the value of the floating leg equals par, and the time–t value of the floating leg is $D(t, T_m)$, where $D(t, T)$ is the time–t price of a zero-coupon bond maturing at time T. The

[9] We assume that both the fixed and floating legs pay \$1 principal at T_n.

time–t value of the fixed leg is equal to $D(t, T_n) + A_{m,n}(t)$, where $A_{m,n}(t) \equiv \sum_{j=m+1}^{n} D(t, T_j)$ is the present value of an annuity associated with the fixed leg of the forward swap contract, also known as the "price value of the basis point" (PVBP) of a swap. The time-t forward swap rate, $S_{m,n}(t)$, is the rate on the fixed leg that makes the present value of the swap contract equal to zero at t:

$$S_{m,n}(t) = \frac{D(t, T_m) - D(t, T_n)}{A_{m,n}(t)}. \tag{26}$$

This forward swap rate becomes the spot swap rate $S_{m,n}(T_m)$ at time T_m.

A swaption gives its holder the right but not the obligation to enter into an interest rate swap either as a fixed leg (payer swaption) or as a floating leg (receiver swaption) with a pre-specified fixed coupon rate. The underlying security of a swaption is a forward start interest rate swap contract. For example, let T_m be the expiration date of the swaption, K be the coupon rate on the swap, and T_n be the final maturity date of the swap. The payer swaption allows the holder to enter into a swap at time T_m with a remaining term of $T_n - T_m$ and to pay the fixed annuity of K. At time t, this swaption is usually called a $(T_m - t)$ into $(T_n - T_m)$ payer swaption, also known as a $(T_m - t)$ by $(T_n - T_m)$ payer swaption, where $(T_m - t)$ is the option maturity and $(T_n - T_m)$ is the tenor of the underlying swap. Because the value of the floating leg will be par at time T_m, the payer swaption is equivalent to a put option on a bond with a coupon rate K and a remaining maturity of $T_n - T_m$, where the strike of this put option is \$1. Similarly, the receiver swaption is equivalent to a call option on the same coupon bond with the strike price of \$1.

Let $\mathcal{P}_{m,n}(t; K)$ and $\mathcal{R}_{m,n}(t; K)$ denote the time-t value of a European payer and receiver swaption, respectively, expiring at T_m with strike K on a forward start swap for the time period between T_m and T_n. At the option expiration date T_m, the payer swaption has a payoff of

$$[1 - D(T_m, T_n) - K A_{m,n}(T_m)]^+ = A_{m,n}(T_m)[S_{m,n}(T_m) - K]^+,$$

where equation (26) evaluated at T_m is used. Therefore, the time-t ($< T_m$) price of this payer swaption is given by

$$\begin{aligned}\mathcal{P}_{m,n}(t;K) &= \mathbb{E}_t^{\mathbb{Q}}\left\{e^{-\int_t^{T_m} r(s)ds} A_{m,n}(T_m)\left[S_{m,n}(T_m)-K\right]^{+}\right\} \\ &= A_{m,n}(t)\,\mathbb{E}_t^{\mathbb{A}^{m,n}}\left\{\left[S_{m,n}(T_m)-K\right]^{+}\right\}, \end{aligned} \tag{27}$$

where $\mathbb{Q}$ is the risk-neutral measure and $\mathbb{A}^{m,n}$ is the annuity measure with $A_{m,n}(t)$ as the numeraie. That is, the Radon-Nikodym derivative of the annuity measure with respect to the risk-neutral measure is $\frac{d\mathbb{A}^{m,n}}{d\mathbb{Q}} = e^{-\int_t^{T_m} r(s)ds}\frac{A_{m,n}(T_m)}{A_{m,n}(t)}$. Similarly, the time-$t$ price of the receiver swaption is given by

$$\mathcal{R}_{m,n}(t;K) = A_{m,n}(t)\,\mathbb{E}_t^{\mathbb{A}^{m,n}}\left\{\left[K-S_{m,n}(T_m)\right]^{+}\right\}. \tag{28}$$

We note from (27) and (28) that a swaption is tied to two sources of uncertainty: (i) the forward swap rate $S_{m,n}(t)$, and (ii) the swap's PVBP realized at time T_m, $A_{m,n}(T_m)$. The change of measure from $\mathbb{Q}$ to $\mathbb{A}^{m,n}$ allows us to focus on the risk of $S_{m,n}(t)$ and facilitates the pricing of swaptions.

4.2 Measure of swaption-implied variance risk premium

Variance swaps on equities allow one to hedge the risk of the realized variance of stock returns. The *variance contract on swap rates* that we develop below allows us to hedge the risk of the realized variance of interest rate swap rates. At time t, the short leg promises to pay the long leg at T_m:

$$A_{m,n}(T_m)\left[\left(\ln\frac{S_{m,n}(t+\Delta)}{S_{m,n}(t)}\right)^2+\left(\ln\frac{S_{m,n}(t+2\Delta)}{S_{m,n}(t+\Delta)}\right)^2+\cdots+\left(\ln\frac{S_{m,n}(T_m)}{S_{m,n}(T_m-\Delta)}\right)^2\right], \tag{29}$$

the product of the realized variance of the log forward swap rate $\log S_{m,n}(t)$ over $[t, T_m]$ and the PVBP $A_{m,n}(T_m)$. In return, the long leg pays the short leg $A_{m,n}(T_m)\times\mathbb{VP}_{m,n}(t)$ at T_m, where $\mathbb{VP}_{m,n}(t)$ is determined at time t such that the value of the contract equals zero at initiation. We refer to $\mathbb{VP}_{m,n}(t)$ as the variance price of the forward swap rate.

The *variance contract on swap rates* uses the sum of squared log changes to measure the realized variance of forward swap rates over $[t, T_m]$. Similar to the payoff of a swaption, the payoff of the *variance contract on swap rates* depends on the realized variance of forward swap rates as well as an annuity discount factor. This design makes it convenient to obtain the variance price $\mathbb{VP}_{m,n}(t)$ by a change of the risk-neutral measure to the corresponding annuity measure. It also makes it easier to replicate the variance contract using swaptions given the similar payoff structures. The variance price $\mathbb{VP}_{m,n}(t)$ is the $\mathbb{A}^{m,n}$-expectation of the quadratic variation of the forward swap rate $S_{m,n}(t)$ over $[t, T_m]$.

Similar to the equity variance swap whose payoff can be replicated using a portfolio of out-of-the-money equity options, the time-varying payoff of the *variance contract on swap rates* can be replicated using a portfolio of out-of-the-money swaptions written on $S_{m,n}(t)$. In particular, generalizing the algorithm used by CBOE in constructing VIX, we have

$$\mathbb{IV}_{m,n}(t) \equiv \frac{2}{A_{m,n}(t)} \left\{ \int_{K>S_{m,n}(t)} \frac{1}{K^2} \mathcal{P}_{m,n}(t;K)dK + \int_{K<S_{m,n}(t)} \frac{1}{K^2} \mathcal{R}_{m,n}(t;K)dK \right\}, \quad (30)$$

where $T_m - t$ is the time-to-maturity. As observed from (30), this replication portfolio contains positions in out-of-the-money swaptions with a weight that is inversely proportional to their strikes. A similar replication portfolio based on equity options has been employed in the literature to construct model-free implied volatility measures (Bollerslev, Tauchen, and Zhou, 2009; Carr and Wu, 2009). The swap-rate variance risk premium is defined as the difference between this ex ante expectation of the future swap rate variation over [t, T_m] under the annuity measure $\mathbb{A}^{m,n}$ and the corresponding expectation under the physical measure over the same time interval,

$$VRP_{m,n}(t) \equiv \mathbb{IV}_{m,n}(t) - \mathbb{RV}_{m,n}(t). \quad (31)$$

5 Data and Estimates

In this section we describe the estimates of the swaptions-based variance risk premium, Treasury yield data, and other control variables used in predictive regressions in Section 6.

5.1 Estimates of swaptions-based variance risk premium

We obtain daily LIBOR rates with maturities of 3, 6, 9, and 12 months, as well as daily 2-, 3-, 4-, 5-, 7-, 10-, 15-, 20-, 25-, 30-, and 35-year spot swap rates between June 1, 1993 and January 31, 2013 from J.P. Morgan. We bootstrap the swap rates to first obtain daily zero-curves. Then we construct the PVBP curve $A_{m,n}(t)$ and forward swap rate curve $S_{m,n}(t)$ up to 35 years according to (26).[10]

Daily observations of (European) swaption prices are combined from J.P. Morgan and Barclays Capital, two of the largest inter-dealer brokers in interest rate derivatives markets. We focus on 1-month expiry swaptions in line with the monthly frequency of our main predictive regressions. In particular, we obtain one-month swaptions on six swap tenors (1, 2, 5, 10, 20, and 30 years). The market convention is to quote swaption prices in terms of their log-normal implied volatility based on Black (1976) formula.[11]

The swaption prices from J.P. Morgan are available between June 1, 1993 and January 31, 2013 with five strikes, including at-the-money-forward (ATMF), ATMF ± 100, and ATMF ± 50 basis points. The swaption prices from Barclays are between December 1, 2004 and January 31, 2013 with thirteen strikes, including ATMF, ATMF ± 200, ATMF ± 150, ATMF ± 100, ATMF ± 75, ATMF ± 50, and ATMF ± 25 basis points. In our empirical analysis, we use the swaption prices from J.P. Morgan from June 1, 1993 through December 1, 2004

[10] We first use a standard cubic spline algorithm to interpolate the swap rates at semiannual intervals from one year to 35 years. We then solve for the zero curve by bootstrapping the interpolated par curve with swap rates as par bond yields. The day count convention is 30/360 for the fixed leg, and Actual/360 for the floating leg.

[11] Many market participants think in terms of normal (or absolute or basis point) implied volatilities–the volatility parameter that, plugged into the normal pricing formula, matches a given price–as they are more uniform across the swaption grid and more stable over time than log-normal implied volatilities.

and those from Barclays after December 1, 2004.[12]

To obtain swaption prices on a continuum of strikes as requested by Eq. (30), following Carr and Wu (2009) and Du and Kapadia (2012), we interpolate implied volatilities across the range of observed strikes and use implied volatility of the lowest (highest) available strike to replace that of the strikes below (above). We further generate 200 implied volatility points equally spaced over a strike range with moneyness between $0.9 \times S_{m,n}(t)$ and $1.1 \times S_{m,n}(t)$, where $S_{m,n}(t)$ is the current forward swap rate on each day.

Finally, as a proxy for physical expectation of swap-rate variance, we use intraday 10-year interest rate swap quotes at five-minute intervals from 8:30 to 15:00 (following Wright and Zhou (2009)) and then fit the HAR model to obtain expectation of the realized variance under physical measure. Figure 3 plots implied variance, expected variance, and resulting swaption-implied variance risk premium.

5.2 Treasury yield data

In our empirical exercise we use Fama-Bliss data set of zero-coupon Treasury yield data from CRSP to compute excess returns of the bonds for two to five-year bonds. The sample period of our study is January 1991 to December 2012, the frequency is monthly. In general, we denote by $r_{t+h}^{(\tau)} = p_{t+h}^{(\tau-h)} - p_t^{(\tau)}$, the $h-$period log return on a $\tau-$year bond with the log price $p_t^{(\tau)}$. The excess bond return is defined as

$$rx_{t+h}^{(\tau)} \equiv r_{t+h}^{(\tau)} - y_t^{(h)}, \tag{32}$$

where $y_t^{(h)}$ is the $h-$period yield. In our application we consider $h = 1, 3$, and 12 months.[13] The summary statistics of the Treasury excess returns is presented in Panel A of Table 2. A notable difference between one-year and one-month returns is that the latter are much less

[12] All our empirical results remain unchanged if we use only the J.P. Morgan swaption data rather than a combination of the J.P. Morgan and Barclays Capital swaption data.

[13] Note that maturities $\tau - 1$ month and $\tau - 3$ months, where $\tau = 2, \ldots, 5$, do not exist so we have obtained prices of these securities via linear interpolation of adjacent maturity prices.

persistent than the former.

5.3 Other predictive variables

Equity variance risk premium As a proxy for a risk-neutral expectation $\mathbb{E}_t^{\mathbb{Q}}(RV_{t+\tau,\tau})$ of return variance, we use monthly data of the squared VIX index – the "model-free" option-implied variance based on the highly liquid S&P 500 index options and the "model-free" approach to compute the risk-neutral variance of a fixed 30-day maturity. The data is obtained from the Chicago Board of Options Exchange (CBOE).[14]

As a proxy for physical expectation of return variance, we first compute the realized variance RV_t following the methodology described by Bollerslev, Tauchen, and Zhou (2009). The realized variance is estimated using tick data from S&P500 futures, one of the most heavily traded assets on the Chicago Mercantile Exchange (CME). The realized variance $RV_{t,\tau}$ is defined as a squared variation between day $t-\tau$ and t with τ being typically a month, or equivalently, 22 days. To estimate the expectation of return variation of the next period $\mathbb{E}_t^{\mathbb{P}}(RV_{t+\tau,\tau})$, we first compute the realized variance RV_t during the day t as a sum of squared deviations of the price changes over the five-minute intervals:

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2, \tag{33}$$

where $r_{t,i} = logP(t-1+\frac{i}{M}) - logP(t-1+\frac{i-1}{M})$ is the intra-day log return in the i^{th} sub-interval of day t and $P(t-1+\frac{i}{M})$ is the asset price at time $t-1+\frac{i}{M}$. Ideally, the sampling frequency for the computation of the realized variance should go to infinity. However, in practice high-frequency data is affected by a number of microstructure issues such as price discreteness, bid-ask spreads, and nonsynchronous trading effects. A number of studies, for example, Andersen, Bollerslev, Diebold, and Labys (2000) and Hansen and

[14]For the computation of the model-free measure of the implied variance see, for example, Demeterfi, Derman, Kamal, and Zou (1999), Britten-Jones and Neuberger (2000), Carr and Madan (2001), Jiang and Tian (2007), and Bollerslev, Gibson, and Zhou (2011).

Lunde (2006) suggest that a five-minute sampling frequency provides a reasonable choice. Thus, our realized variance series is based on the $r_{t,i}$ computed between 9:30 and 16:00 of each trading day at the five-minute intervals. The monthly realized variance is computed by averaging daily variances within 22 trading days over the month, $RV_{t,mon} = \frac{1}{22}\sum_{j=0}^{21} RV_{t-j}$. The weekly realized variation $RV_{t,week}$ is correspondingly defined by the average of the five daily measures, $RV_{t,week} = \frac{1}{5}\sum_{j=0}^{4} RV_{t-j}$.

The expected variance risk premium $\mathbb{E}_t^{\mathbb{P}}(RV_{t+\tau,\tau})$ is based on the forecasting heterogeneous autoregressive model of realized volatility (HAR-RV), suggested by Andersen, Bollerslev, and Diebold (2007) and Corsi (2009). The model is simple to implement yet it produces empirically highly accurate forecast. The model aims to capture the long memory behavior of volatility by incorporating the daily, weekly, and monthly realized variance estimates. HAR-RV model is a parsimonious model of higher-order regressions, where the one-month ahead variance is an affine combination of the previous month daily, weekly, and monthly realized variances:

$$RV_{t+22,mon} = \alpha + \beta_D RV_t + \beta_W RV_{t,week} + \beta_M, RV_{t,mon} + \epsilon_{t+22,mon}. \tag{34}$$

The variance risk premium for both equity and swaptions is computed as a difference between the risk-neutral and physical variance expectations as defined in equation (20). Figures 2 and 3 plot implied variance, expected variance, and resulting variance risk premium for both equity and interest rate swaptions markets, respectively. First result from this figures is that the variance risk premium almost everywhere positive suggesting that market participants seek compensation for variance exposure. Second, market variance risk premium increases during NBER recessions, represented by shaded blue bars. Such an increase in general captures the spirit of increased uncertainty amid recessions: this is a reason that we loosely refer to the variance risk premium a as a compensation for uncertainty.[15] The variance risk premium has also been higher in 1997-1998 period: although this period is not

[15] Bloom (2009) provides a similar argument about the relationship between uncertainty and volatility.

formally marked as a recession, this was a period of high turbulence in the Asian markets and also LTCM collapse. The sample period for swaptions is much shorter but a similar pattern is evident on Figure 3 as well: variance risk premium increased notably during the last financial crisis and amid European crisis in mid-late 2011.

In addition to the variance risk premium, we use two other variables in our predictive regressions. First, we use the classical Fama and Bliss (1987) predictor, forward spread, the spread between the forward rate of a particular maturity and a risk-free rate; Second, we use Cochrane and Piazzesi (2005) factor, an affine combination of forward rates. Both variables are computed using Fama-Bliss data set, downloaded from CRSP.

Panel B of Table 2 summarizes statistics for the predictive variables. It is notable that while forward spread and CP factors are extremely persistent, the variance risk premium is not. The first-order autocorrelation coefficients for the two factors are on the range of 0.91 and 0.97, while the same AR(1) coefficient for the variance risk premium is 0.28. Panel A shows that 1-year excess bond returns have a similar magnitude of persistence as forward spread and CP factors, while 1-month excess returns of Treasury bonds are less persistent.

6 Empirical Results

In this section we discuss how well the variance risk premium – endogenous proxy for economic uncertainty in our model – predicts Treasury bond returns. We first discuss empirical results related to equity variance risk premium and then - to swaptions-based variance risk premium. To assess its predictability content, we run the following regressions:

$$rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)VRP_t + \beta_2^{(\tau)}(h)FS_t^{(\tau)} + CP_t + \epsilon_{t+h}^{(\tau)}, \tag{35}$$

where $rx_{t+h}^{(\tau)}$ is the $h-$period excess return on a $\tau-$year Treasury bond, VRP_t is the variance risk premium, $FS_t^{(\tau)}$ is the $(\tau)-$maturity Fama-Bliss forward spread, and CP is the Cochrane-Piazzesi factor. Excess returns are computed using Fama-Bliss discount bond data set. For each bond maturity ($\tau = 2, 3, 4, 5$ years), and each return horizon ($h = 1, 3, 12$ months) we run individual regressions as well as regressions on subset of factors and also run a kitchen-sink-type regression using all three factors. We consider two cases of the variance risk premium. The first one is the market variance risk premium advocated in Bollerslev, Tauchen, and Zhou (2009), where the authors show that it has some notable predictability for expected stock returns. The second version of the variance risk premium is new and based on the data from the interest rate swaptions market that is relatively young.

6.1 Predictability results with equity variance risk premium

Tables 3, 4, and 5 present regression results for one-month, three-month, and one-year holding period bond excess returns, correspondingly. Table 3 reports that the VRP is significant for one-month excess bond returns for maturities beyond 3 years in a joint regression with a forward spread and CP factors. While statistically significant at the 10% level for 3-year bond excess returns, VRP shows statistical significance at the 5% level in joint regressions with forward spread and CP factors.[16] Although the highest adjusted R^2 is 1.44% for 5-year excess bond returns, the overall result from this table hints that variance risk premium may possibly have some information content relevant for short-horizon variation of bond returns beyond that contained in standard predictors. We do not observe similar kind of significance in Tables 4 and 5 with the exception of some marginally significant VRP for 3-month horizon excess returns on 3-year bonds (Table 4), where we find marginal significance in the presence of CP and forward spread factors. Thus, statistical significance of the market-based VRP diminishes with bond maturity. Overall, the conclusion from these tables is that irrespective of the holding period return, market variance risk premium is only marginally relevant for predicting bond excess returns and that this relevance diminishes with bond maturity. The reason of this result can be seen in Table 2 that shows that excess bond returns, forward spread, and CP factors are extremely persistent but variance risk premium is not. As the

[16] Here in the following tables, standard errors are Newey-West corrected.

investment horizon shortens, from one year to three months to one month, bond returns become less persistent, and, consequently, the variance risk premium starts playing a more important role in predicting bond excess returns. The sign of the variance risk premium beta is always positive, consistent with our theoretical prediction, meaning that investors seek compensation at the times of the heightened volatility. The variance risk premium is statistically significant in the presence of either forward spread or CP factor, or both. These results suggest that the variance risk premium captures bond return variation at shorter horizons, while standard predictors are more important at longer horizons. These results are broadly consistent with Mueller, Vedolin, and Zhou (2011) who also document that the market-based variance risk premium has the strong predictive power for bond returns in the short-run (1-month), that disappears in the long-run (1-year).

6.2 Predictability with swaptions-implied variance risk premium

Tables 6, 7, and 8 report predictability results of the variance risk premium derived from interest rate swaptions for 1-, 3-, and 12-month Treasury excess returns. Table 6 reports the results for 1-month excess returns. The difference of this table's results with those in Table 3 is quite stark. First observation is that swaptions-based variance risk premium (SVRP) is strongly significant in the univariate regressions at the 1% level of statistical significance and adjusted R^2 varies from 29 percent to 21 percent and declines with the maturity (column 1 of Table 6). Second observation is that it is significant in the presence of Fama-Bliss factor (column 4) and Cochrane-Piazzessi factor (column 5), and still highly significant in the multi-variate regression with both factors (column 6). Third observation is that the SVRP in the predictive regressions seems to add nontrivial forecasting power: for example, when it is added to a CP factor, the adjusted R^2 increases from 28 percent to 41 percent for 2-year returns; when it is added to a Fama-Bliss factor, the adjusted R^2 increases from 54 to 68 percent. An increase in R^2 is similar albeit slightly lower (especially in the case of a Fama-Bliss factor) as the bond maturity increases. The take-away from Table 6 is that derivatives-based variance risk premium seems to have information useful for predicting expected bond returns beyond that contained in the standard predictors. We contrast this result with market-based variance risk premium, which does not provide such evidence (see Table 3).

Turning to the predictability results of the 3-month holding period Treasury excess returns, reported in Table 7, we first observe that the SVRP is still statistically significant at the 1% level for short-maturity (e.g. 2-year) bonds, however, this statistical significance decreases to 5% significance and then to marginal or no significance as bond maturity increases. Second, SVRP still has some non-trivial contribution to the predictability of excess returns. For example, in the multivariate regressions with a CP factor, adjusted R^2 increases by 6 percent – from 27 to 33 percent – but that contribution falls as bond's maturity increases. Third, in all but 2-year maturity bond, the statistical significance of the SVRP is marginal or non-existent in the presence of other predictors. Comparing results from Tables 6 and 7 it seems that swaptions-based variance risk premium captures the short-run risks in the Treasury excess returns. Indeed, SVRP seems to be capturing variation in the short-horizon expected returns, where significance of such contribution is declining with bond's maturity. This predictability survives in the presence of other predictors, which remain important at longer horizons. Table 8 just confirms this pattern: 1-year returns are much less predictable by the SVRP then 1-month or 3-month returns.

7 Concluding remarks

We study bond pricing implications in the context of our proposed long-run risk asset-pricing model with uncertainty risks and inflation. We show that our model is promising in explaining the first and second moments of the bond market. First, we show that the long-run risk factor is crucial in fitting the level of the interest rates. Second, we study the variation in bond short-horizon and long-horizon returns, predictability patterns that were documented separately in earlier literature. Our empirical results indicate that swaptions-based variance risk premium drives short-horizon (one- and three-month) Treasury bond returns, while other popular predictive variables, such as Fama-Bliss forward spread or Cochrane-Piazzessi factor drive the variation in the long-horizon (one-year) Treasury bond returns. In the model time-varying bond risk premium is driven by the variance of the endowment growth process and the volatility-of-volatility (uncertainty) of endowment process. Since variance risk premium in the model loads entirely on the vol-of-vol factor, we interpret short-horizon variation of the bond returns as due to the vol-of-vol factor, whereas long-horizon variation – to the variance of the endowment growth factor. Thus, our model allows to reconcile separate empirical findings about bond returns.

A Appendix

A.1 Solution for the consumption-wealth ratio coefficients

We solve for A_0, A_x, A_{σ^2}, A_q - state variables' loadings in the price-consumption ratio z_t. We solve for A's by pricing $r_{c,t+1}$ using Euler equation (10), wealth return equation (7) and assumed z dynamics in equation (6). Thus, Euler equation becomes:

$$\mathbb{E}_t\left[\exp(m_{t+1} + r_{c,t+1})\right] = \mathbb{E}_t\left[\exp\left(\theta \ln\delta - \frac{\theta}{\psi} g_{t+1} + \theta r_{c,t+1}\right)\right] = 1. \tag{36}$$

Using Jensen's inequality, obtain:

$$\mathbb{E}_t\left[\theta \ln\delta - \frac{\theta}{\psi} g_{t+1} + \theta r_{c,t+1}\right] + \frac{1}{2}\text{Var}_t\left[\theta \ln\delta - \frac{\theta}{\psi} g_{t+1} + \theta r_{c,t+1}\right] = 0. \tag{37}$$

Substituting out $r_{c,t+1}$, z_{t+1}, and z_t, obtain:

$$\begin{aligned}
&\mathbb{E}_t[\theta \ln\delta - \frac{\theta}{\psi}(\mu_g + x_t + \sigma_{g,t} z_{g,t+1}) + \theta(\kappa_0 + \kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) - \\
&A_0 - A_x x_t - A_\sigma \sigma^2_{g,t} - A_q q_t + \mu_g + x_t + \sigma_{g,t} z_{g,t+1})] + \\
&\frac{1}{2}\text{Var}_t[\theta \ln\delta - \frac{\theta}{\psi}(\mu_g + x_t + \sigma_{g,t} z_{g,t+1}) + \theta(\kappa_0 + \kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{g,t+1} + A_q q_{t+1}) - \\
&A_0 - A_x x_t - A_\sigma \sigma^2_{g,t} - A_q q_t + \mu_g + x_t + \sigma_{g,t} z_{g,t+1})] = 0.
\end{aligned} \tag{38}$$

To solve for A_x, match terms in front of x_t:

$$-\frac{\theta}{\psi} + \theta(\kappa_1 A_x \rho_x - A_x + 1) = 0 \quad \Rightarrow \quad A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_x}. \tag{39}$$

To solve for A_σ, match terms in front of $\sigma^2_{g,t}$:

$$\begin{aligned}
&(\theta\kappa_1 A_\sigma\rho_\sigma - \theta A_\sigma)\sigma^2_{g,t} + \frac{1}{2}\mathrm{Var}_t\left[-\frac{\theta}{\psi}\sigma^2_{g,t}z_{g,t+1} + \theta\kappa_1 A_x\phi_e\sigma_{g,t}z_{x,t+1} + \theta\sigma_{g,t}z_{g,t+1}\right] = \\
&\theta A_\sigma(\kappa_1\rho_\sigma - 1)\sigma^2_{g,t} + \frac{1}{2}\mathrm{Var}_t\left[\left(\theta - \frac{\theta}{\psi}\right)\sigma_{g,t}z_{g,t+1} + \theta\kappa_1 A_x\phi_e\sigma_{g,t}z_{x,t+1}\right] = 0 \quad \Rightarrow \\
&\theta A_\sigma(\kappa_1\rho_\sigma - 1) + \frac{1}{2}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta\kappa_1 A_x\phi_e)^2\right] = 0 \quad \Rightarrow \\
&A_\sigma = \frac{1}{2\theta(1-\kappa_1\rho_\sigma)}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + (\theta\kappa_1 A_x\phi_e)^2\right].
\end{aligned} \tag{40}$$

To solve for A_0, set constant terms under the expectation in (38) equal to zero:

$$\begin{aligned}
\theta\ln\delta + \theta(\kappa_0 + \kappa_1(A_0 + A_\sigma a_\sigma + A_q a_q)) - A_0 + \left(\theta - \frac{\theta}{\psi}\right)\mu_g = 0 \quad \Rightarrow \\
A_0 = \frac{1}{1-\kappa_1}\left[\ln\delta + \kappa_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q) + \left(1 - \frac{1}{\psi}\right)\mu_g\right].
\end{aligned} \tag{41}$$

To solve for A_q, match terms in front of q_t and set equal to zero:

$$\begin{aligned}
&(\theta\kappa_1 A_q\rho_q - \theta A_q)q_t + \frac{1}{2}\mathrm{Var}_t[\theta\kappa_1 A_\sigma\sqrt{q_t}z_{\sigma_{t+1}} + \theta\kappa_1 A_q(\rho_q q_t + \phi_q\sqrt{q_t}z_{q_{t+1}}) - \theta A_q q_t] = \\
&\theta A_q(\kappa_1\rho_q - 1)q_t + \frac{1}{2}\mathrm{Var}(\theta\kappa_1 A_\sigma\sqrt{q_t}z_{\sigma_{t+1}} + \theta\kappa_1 A_q\phi_q\sqrt{q_t}z_{q_{t+1}}) = 0 \quad \Longrightarrow \\
&\frac{1}{2}(\theta\kappa_1\phi_q)^2 A_q^2 + \theta(\kappa_1\rho_q - 1)A_q + \frac{1}{2}(\theta\kappa_1 A_\sigma)^2 = 0 \quad \text{or, equivalently,} \\
&(\theta\kappa_1\phi_q)^2 A_q^2 + 2\theta(\kappa_1\rho_q - 1)A_q + (\theta\kappa_1 A_\sigma)^2 = 0.
\end{aligned} \tag{42}$$

The solution for A_q represents the solution to a quadratic equation and is given by:

$$A_q^{\pm} = \frac{1-\kappa_1\rho_q \pm \sqrt{(1-\kappa_1\rho_q)^2 - (\theta\kappa_1^2\phi_q A_\sigma)^2}}{\theta(\kappa_1\phi_q)^2}. \tag{43}$$

As Tauchen (2011) notes, a "positive" root A_q^+ has an unfortunate property that

$$\lim_{\phi_q\to 0}\phi_q^2 A_q^+ \neq 0, \tag{44}$$

which is, essentially, a violation of the transversality condition in this setting: though uncertainty q_t vanishes with $\phi_q \to 0$, the effect of it on prices is not. Therefore, we choose A_q^- root as a viable solution for A_q:

$$A_q = \frac{1 - \kappa_1 \rho_q - \sqrt{(1 - \kappa_1 \rho_q)^2 - \theta^2 \kappa_1^4 \phi_q^2 A_\sigma^2}}{\theta (\kappa_1 \phi_q)^2}. \tag{45}$$

To insure that determinant in (45) is positive, we also need to impose a constraint on the magnitude of the shock $z_{q,t+1}$:

$$\phi_q^2 \leq \frac{(1 - \kappa_1 \rho_q)^2}{\theta^2 \kappa_1^4 A_\sigma^2}. \tag{46}$$

A.2 Solution for the pricing kernel

Using the solutions for A's obtained in A.1, we solve for the expected value $\mathbb{E}_t(m_{t+1})$ and variance $\mathrm{Var}_t(m_{t+1})$ of the pricing kernel:

$$\begin{aligned}
&\mathbb{E}_t[m_{t+1}] = \theta \ln \delta - \frac{\theta}{\psi} \mathbb{E}_t[g_{t+1}] + (\theta - 1)\mathbb{E}_t[r_{c,t+1}] = \\
&= \theta \ln \delta - \frac{\theta}{\psi}(\mu_g + x_t) + (\theta - 1)\mathbb{E}_t(\kappa_0 + \kappa_1 z_{t+1} + g_t - z_t) \\
&= \theta \ln \delta - \frac{\theta}{\psi}(\mu_g + x_t) + (\theta - 1)[\kappa_0 + \kappa_1(A_0 + A_x \rho_x x_t + A_\sigma(a_\sigma + \rho_\sigma \sigma_{g,t}^2) + A_q(a_q + \rho_q q_t)) \\
&+ \mu_g + x_t - A_0 - A_x x_t - A_\sigma \sigma_{g,t}^2 - A_q q_t] \\
&= \theta \ln \delta + \underbrace{\left((\theta - 1) - \frac{\theta}{\psi}\right)}_{-\gamma} \mu_g + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\
&- \frac{\theta}{\psi} x_t + (\theta - 1)[(A_x(\kappa_1 \rho_x - 1) + 1)x_t + A_\sigma(\kappa_1 \rho_\sigma - 1)\sigma_{g,t}^2 + A_q(\kappa_1 \rho_q - 1)q_t] \\
&= \theta \ln \delta - \gamma(\mu_g + x_t) + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] \\
&+ (\theta - 1)[A_x(\kappa_1 \rho_x - 1)x_t + A_\sigma(\kappa_1 \rho_\sigma - 1)\sigma_{g,t}^2 + A_q(\kappa_1 \rho_q - 1)q_t].
\end{aligned} \tag{47}$$

The variance of the SDF $\text{Var}_t[m_{t+1}]$ is given by

$$
\begin{aligned}
&\text{Var}_t[m_{t+1}] = \text{Var}_t\left[\theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta-1) r_{c,t+1}\right] \\
&= \text{Var}_t\left[-\frac{\theta}{\psi} g_{t+1} + (\theta-1)\kappa_1(A_0 + A_x x_{t+1} + A_\sigma \sigma^2_{t+1} + A_q q_{t+1})\right] \\
&= \text{Var}_t\left[-\frac{\theta}{\psi}\sigma_{g,t} z_{g,t+1} + (\theta-1)[\kappa_1(A_x \phi_e \sigma_{g,t} z_{x,t+1} + A_\sigma \sqrt{q_t} z_{\sigma,t+1} + A_q \phi_q \sqrt{q_t} z_{q,t+1}) + \sigma_{g,t} z_{g,t+1}]\right] \\
&= \text{Var}_t\left[\left((\theta-1) - \frac{\theta}{\psi}\right)\sigma_{g,t} z_{g,t+1} + (\theta-1)\kappa_1(A_x \phi_e \sigma_{g,t} z_{x,t+1} + A_\sigma \sqrt{q_t} z_{\sigma,t+1} + A_q \phi_q \sqrt{q_t} z_{q,t+1})\right] \\
&= \gamma^2 \sigma^2_{g,t} + (\theta-1)^2 \kappa_1^2 \left[A_x^2 \phi_e^2 \sigma^2_{g,t} + (A_\sigma^2 + A_q^2 \phi_q^2) q_t\right].
\end{aligned}
\tag{48}
$$

A.3 Solution for the $n-$period real bond price

In this section we derive the (log) price of an $n-$period bond in closed form. A general recursion for solving for the $n-$period bond prices is as follows:

$$
P_t^n = \mathbb{E}_t\left[M_{t+1} P_{t+1}^{n-1}\right]. \tag{49}
$$

Then the $n-$period log bond price

$$
p_t^n = \mathbb{E}_t[m_{t+1}] + \frac{1}{2}\text{Var}_t[m_{t+1}] + \mathbb{E}_t[p_{t+1}^{n-1}] + \frac{1}{2}\text{Var}_t[p_{t+1}^{n-1}] + \text{Cov}_t[m_{t+1}, p_{t+1}^{n-1}]. \tag{50}
$$

Assuming that p_t^n follows the affine representation of the state variables:

$$
p_{t+1}^{n-1} = B_0^{n-1} + B_1^{n-1} x_{t+1} + B_2^{n-1} \sigma^2_{t+1} + B_3^{n-1} q_{t+1}. \tag{51}
$$

The first two terms in (50) are given in (47) and (48). The last three terms in (50) are computed using a pricing conjecture (51). Thus, the expectation term for the bond price is:

$$\begin{aligned}\mathbb{E}_t\left[p_{t+1}^{n-1}\right] =& B_0^{n-1} + B_1^{n-1}\mathbb{E}_t[x_{t+1}] + B_2^{n-1}\mathbb{E}_t[\sigma_{t+1}^2] + B_3^{n-1}\mathbb{E}_t[q_{t+1}] \\ &= B_0^{n-1} + B_1^{n-1}\rho_x x_t + B_2^{n-1}(a_\sigma + \rho_\sigma \sigma_t^2) + B_3^{n-1}(a_q + \rho_q q_t) \\ &= (B_0^{n-1} + B_2^{n-1}a_\sigma + B_3^{n-1}a_q) + B_1^{n-1}\rho_x x_t + B_{n-1}^2 \rho_\sigma \sigma_t^2 + B_3^{n-1}\rho_q q_t,\end{aligned} \tag{52}$$

and the variance term is:

$$\mathrm{Var}_t\left[p_{t+1}^{n-1}\right] = \mathbb{E}_t\left[p_{t+1}^{n-1} - \mathbb{E}_t\left[p_{t+1}^{n-1}\right]\right]^2 = \left(B_1^{n-1}\phi_e\right)^2 \sigma_{g,t}^2 + \left(\left(B_2^{n-1}\right)^2 + \left(B_3^{n-1}\phi_q\right)^2\right) q_t. \tag{53}$$

Last, express the covariance term as a function of the state variables:

$$\begin{aligned}\mathrm{Cov}_t\left[p_{t+1}^{n-1}, m_{t+1}\right] &= \mathbb{E}_t\left[\left(p_{t+1}^{n-1} - \mathbb{E}_t\left[p_{t+1}^{n-1}\right]\right) \times \left(m_{t+1} - \mathbb{E}_t\left[m_{t+1}\right]\right)\right] \\ &= \mathbb{E}_t\left[\left(B_1^{n-1}\phi_e\sigma_{g,t}z_{x,t+1} + B_2^{n-1}\sqrt{q_t}z_{\sigma,t+1} + B_3^{n-1}\phi_q\sqrt{q_t}z_{q,t+1}\right)\right. \\ &\quad \left.\times\left((1-\gamma)\sigma_{g,t}z_{g,t+1} + (\theta-1)\kappa_1(A_x\phi_e\sigma_{g,t}z_{x,t+1} + A_\sigma\sqrt{q_t}z_{\sigma,t+1} + A_q\phi_q\sqrt{q_t}z_{q,t+1})\right)\right] \\ &= (\theta-1)\kappa_1\left[A_x B_1^{n-1}\phi_e^2\sigma_{g,t}^2 + A_\sigma B_2^{n-1}q_t + A_q B_3^{n-1}\phi_q^2 q_t\right].\end{aligned} \tag{54}$$

Write down p_t^n as a sum of (47), (48), (52), (53), and (54) and collect together constant terms and loadings for state variables x_t, σ_t^2, and q_t. This implies for coefficients:

$$\begin{aligned}B_0^n &= c_0 + B_0^{n-1} + B_2^{n-1}a_\sigma + B_3^{n-1}a_q \\ B_1^n &= c_1 + B_1^{n-1}\rho_x \\ B_2^n &= c_2 - \frac{1}{2}(\theta-1)^2\kappa_1^2 A_x^2\phi_e^2 + B_2^{n-1}\rho_\sigma + \frac{1}{2}\phi_e^2\left[(\theta-1)\kappa_1 A_x + B_1^{n-1}\right]^2 \\ B_3^n &= c_3 + B_3^{n-1}\rho_q + \frac{1}{2}\left[\left(B_2^{n-1}\right)^2 + \left(B_3^{n-1}\phi_q\right)^2\right] + (\theta-1)\kappa_1\left(A_\sigma B_2^{n-1} + A_q B_3^{n-1}\phi_q^2\right),\end{aligned} \tag{55}$$

where c_i, $i = 0, \ldots, 3$ are given by:

$$
\begin{aligned}
c_0 &= \theta \ln \delta - \gamma \mu_g + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)];\\
c_1 &= -\gamma + (\theta - 1)A_x(\kappa_1 \rho_x - 1);\\
c_2 &= \frac{1}{2}\gamma^2 + \frac{1}{2}(\theta - 1)^2 \kappa_1^2 A_x^2 \phi_e^2 + (\theta - 1)A_\sigma(\kappa_1 \rho_\sigma - 1);\\
c_3 &= \frac{1}{2}(\theta - 1)^2 \kappa_1^2 (A_\sigma^2 + A_q^2 \phi_q^2) + (\theta - 1)A_q(\kappa_1 \rho_q - 1).
\end{aligned} \tag{56}
$$

Note that c_i, $i = 0, \ldots, 3$, coefficients are the steady-state loadings for the real risk-free rate:

$$
r_f = -[c_0 \ c_1 \ c_2 \ c_3] \times [1 \ \mathbb{E}_x \ \mathbb{E}_{\sigma^2} \ \mathbb{E}_q]', \tag{57}
$$

A.4 Solution for the nominal risk-free rate

Here we provide a derivation for the nominal risk-free rate, the (negative) of the (log) price of the one period nominal bond. We express the nominal risk-free rate in log terms, similar to the real risk free rate given in equation (13):

$$
\begin{aligned}
r_{f,t}^{\$} &= -\mathbb{E}_t\left[m_{t+1}^{\$}\right] - \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}^{\$}\right]\\
&= -\mathbb{E}_t\left[m_{t+1} - \pi_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left[\pi_{t+1}\right] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}]\\
&= r_{f,t} + \mathbb{E}_t[\pi_{t+1}] - \frac{1}{2}\mathrm{Var}_t[\pi_{t+1}] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}]\\
&= r_{f,t} + a_\pi + \rho_\pi \pi_t - \frac{1}{2}[\phi_\pi^2 + \phi_{\pi g}^2 \sigma_{g,t}^2 + \phi_{\pi\sigma}^2 q_t] + \mathrm{Cov}_t[m_{t+1}, \pi_{t+1}].
\end{aligned} \tag{58}
$$

we need to compute the last term in (58) to complete the expression for the nominal risk-free rate in closed form:

$$
\mathrm{Cov}_t[m_{t+1}, \pi_{t+1}] = \mathbb{E}_t[[m_{t+1} - \mathbb{E}_t[m_{t+1}]] \times [\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}]]]. \tag{59}
$$

The deviations of pricing kernel m_{t+1} and inflation π_{t+1} are given by:

$$\begin{aligned}
m_{t+1} - \mathbb{E}_t[m_{t+1}] &= -\gamma\sigma_{g,t}z_{g,t+1} + (\theta-1)\kappa_1(A_x\phi_e z_{x,t+1} + A_\sigma\sqrt{q_t}z_{\sigma,t+1} + A_q\phi_q\sqrt{q_t}z_{q,t+1}) \\
\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}] &= \phi_\pi z_{\pi,t+1} + \phi_{\pi g}\sigma_{g,t}z_{g,t+1} + \phi_{\pi\sigma}\sqrt{q_t}z_{\sigma,t+1},
\end{aligned} \tag{60}$$

which implies for (59):

$$\mathbb{E}_t[[m_{t+1} - \mathbb{E}_t[m_{t+1}]] \times [\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}]]] = -\gamma\phi_{\pi g}\sigma^2_{g,t} + (\theta-1)\kappa_1 A_\sigma\phi_{\pi\sigma}q_t. \tag{61}$$

Combining together (14), (58), and (61), obtain the closed-form expression for the nominal risk-free rate:

$$\begin{aligned}
r^{\$}_{f,t} &= -\theta\ln\delta + \gamma\mu_g + a_\pi - (\theta-1)[\kappa_0 + (\kappa_1-1)A_0 + \kappa_1(A_\sigma a_\sigma + A_q a_q)] - \frac{1}{2}\phi^2_\pi \\
&+ [\gamma - (\theta-1)A_x(\kappa_1\rho_x - 1)]\,x_t \\
&+ \left[-(\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) - \frac{1}{2}\gamma^2 - \frac{1}{2}(\theta-1)^2(\kappa_1 A_x\phi_e)^2 - \frac{1}{2}\phi^2_{\pi g} - \gamma\phi_{\pi g}\right]\sigma^2_{g,t} \\
&+ \left[-(\theta-1)A_q(\kappa_1\rho_q - 1) - \frac{1}{2}(\theta-1)^2\kappa_1^2(A^2_\sigma + A^2_q\phi^2_q) - \frac{1}{2}\phi^2_{\pi\sigma} + (\theta-1)\kappa_1 A_\sigma\phi_{\pi\sigma}\right]q_t \\
&+ \rho_\pi\pi_t.
\end{aligned} \tag{62}$$

Similarly to the real risk-free rate, the steady-state nominal risk-free rate can be written as follows:

$$r^{\$}_f = -[c^{\$}_0\ c^{\$}_1\ c^{\$}_2\ c^{\$}_3\ c^{\$}_4] \times [1\ \mathbb{E}_x\ \mathbb{E}_{\sigma^2}\ \mathbb{E}_\pi]', \tag{63}$$

where the $c_i^{\$}$, $i = 0, \ldots, 4$ loadings:

$$\begin{aligned} c_0^{\$} &= c_0 - a_\pi + \frac{1}{2}\phi_\pi^2; \\ c_1^{\$} &= c_1; \\ c_2^{\$} &= c_2 + \frac{1}{2}\phi_{\pi g}^2 + \gamma\phi_{\pi g}; \\ c_3^{\$} &= c_3 + \frac{1}{2}\phi_{\pi\sigma}^2 - (\theta - 1)\kappa_1 A_\sigma \phi_{\pi\sigma}; \\ c_4^{\$} &= \rho_\pi. \end{aligned} \tag{64}$$

A.5 Solution for the $n-$period nominal bond price

The $n-$period nominal log bond price $p_t^{\$,n}$ is given by:

$$p_t^{\$,n} = \mathbb{E}_t[m_{t+1}^{\$}] + \frac{1}{2}\text{Var}_t[m_{t+1}^{\$}] + \mathbb{E}_t[p_{t+1}^{\$,n-1}] + \frac{1}{2}\text{Var}_t[p_{t+1}^{\$,n-1}] + \text{Cov}_t[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}]. \tag{65}$$

Assume that $p_t^{\$,n}$ follows the same affine function representation, as in the case of real bonds, with the additional state variable for inflation:

$$p_t^{\$,n} = B_0^{\$,n} + B_1^{\$,n} x_t + B_2^{\$,n} \sigma_t^2 + B_3^{\$,n} q_t + B_4^{\$,n} \pi_t, \tag{66}$$

We know the first and the second terms in (65) from nominal risk-free rate calculations. Compute the last three terms using a pricing conjecture (66):

$$\begin{aligned} \mathbb{E}_t\left[p_{t+1}^{\$,n-1}\right] &= B_0^{\$,n-1} + B_1^{\$,n-1}\rho_x x_t + B_2^{\$,n-1}(a_\sigma + \rho_\sigma \sigma_{g,t}^2) + B_3^{\$,n-1}(a_q + \rho_q q_t) + B_4^{\$,n-1}(a_\pi + \rho_\pi \pi_t) \\ &= \left[B_0^{\$,n-1} + B_2^{\$,n-1} a_\sigma + B_3^{\$,n-1} a_q + B_4^{\$,n-1} a_\pi\right] \\ &+ B_1^{\$,n-1}\rho_x x_t + B_2^{\$,n-1}\rho_\sigma \sigma_{g,t}^2 + B_3^{\$,n-1}\rho_q q_t + B_4^{\$,n-1}\rho_\pi \pi_t. \end{aligned} \tag{67}$$

The shock to the nominal bond price is given by:

$$
\begin{aligned}
p_{t+1}^{\$,n-1} - \mathbb{E}_t\left[p_{t+1}^{\$,n-1}\right] &= B_1^{\$,n-1}\phi_e\sigma_{g,t}z_{x,t+1} + B_2^{\$,n-1}\sqrt{q_t}z_{\sigma,t+1} + B_3^{\$,n-1}\phi_q\sqrt{q_t}z_{q,t+1} \\
&+ B_4^{\$,n-1}[\phi_\pi z_{\pi,t+1} + \phi_{\pi g}\sigma_{g,t}z_{g,t+1} + \phi_{\pi\sigma}\sqrt{q_t}z_{\sigma,t+1}].
\end{aligned}
\tag{68}
$$

Thus, the variance of the nominal bond price - the fourth term in (65) - is given by:

$$
\begin{aligned}
\text{Var}[p_{t+1}^{\$,n-1}] = \mathbb{E}_t\left[p_{t+1}^{\$,n-1} - \mathbb{E}_t\left[p_{t+1}^{\$,n-1}\right]\right]^2 &= \left[(B_1^{\$,n-1}\phi_e)^2 + (B_4^{\$,n-1}\phi_{\pi g})^2\right]\sigma_{g,t}^2 \\
+ \left[\left(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}\right)^2 + \left(B_3^{\$,n-1}\phi_q\right)^2\right] q_t &+ \left(B_4^{\$,n-1}\phi_\pi\right)^2.
\end{aligned}
\tag{69}
$$

Lastly, compute covariance between between nominal pricing kernel $m_{t+1}^{\$}$ and the nominal bond price $p_{t+1}^{\$,n-1}$:

$$
\text{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] = \mathbb{E}_t\left[\left[m_{t+1}^{\$} - \mathbb{E}_t\left[m_{t+1}^{\$}\right]\right] \times \left[p_{t+1}^{\$,n-1} - \mathbb{E}_t\left[p_{t+1}^{\$,n-1}\right]\right]\right],
\tag{70}
$$

where the shock to the nominal pricing kernel in terms of state variables is:

$$
\begin{aligned}
&m_{t+1}^{\$} - \mathbb{E}_t\left[m_{t+1}^{\$}\right] = m_{t+1} - \mathbb{E}_t m_{t+1} - (\pi_{t+1} - \mathbb{E}_t\pi_{t+1}) \\
&= -\gamma\sigma_{g,t}z_{g,t+1} + (\theta-1)\kappa_1\left(A_x\phi_e\sigma_{g,t}z_{x,t+1} + A_\sigma\sqrt{q_t}z_{\sigma,t+1} + A_q\phi_q\sqrt{q_t}z_{q,t+1}\right) \\
&- \phi_\pi z_{\pi,t+1} - \phi_{\pi g}\sigma_{g,t}z_{g,t+1} - \phi_{\pi\sigma}\sqrt{q_t}z_{\sigma,t+1},
\end{aligned}
\tag{71}
$$

and the shock to the nominal bond price, $p_{t+1}^{\$,n-1} - \mathbb{E}_t\left[p_{t+1}^{\$,n-1}\right]$, is given in (68). Thus, a final expression for a covariance term in (65) is:

$$
\begin{aligned}
&\text{Cov}_t\left[m_{t+1}^{\$}, p_{t+1}^{\$,n-1}\right] = \left[(\theta-1)\kappa_1 A_x B_1^{\$,n-1}\phi_e^2 - (\gamma+\phi_{\pi g})B_4^{\$,n-1}\phi_{\pi g}\right]\sigma_{g,t} \\
&+ \left[((\theta-1)\kappa_1 A_\sigma - \phi_{\pi\sigma})(B_2^{\$,n-1} + B_4^{\$,n-1}\phi_{\pi\sigma}) + (\theta-1)\kappa_1 A_q B_3^{\$,n-1}\phi_q^2 q_t\right] q_t \\
&- B_4^{\$,n-1}\phi_\pi^2.
\end{aligned}
\tag{72}
$$

Combining together (58), (67), (69), and (72), obtain the solution for the nominal $n-$period bond price:

$$
\begin{aligned}
B_0^{\$,n} &= c_0 - a_\pi + \left[B_0^{\$,n-1} + B_2^{\$,n-1} a_\sigma + B_3^{\$,n-1} a_q + B_4^{\$,n-1} a_\pi\right] + \frac{1}{2}\phi_\pi^2 \left(B_4^{\$,n-1} - 1\right)^2 \\
B_1^{\$,n} &= c_1 + B_1^{\$,n-1}\rho_x \\
B_2^{\$,n} &= B_2^{\$,n-1}\rho_\sigma + (\theta-1)A_\sigma(\kappa_1\rho_\sigma - 1) + \frac{1}{2}(\gamma + \phi_{\pi g})^2 + \frac{1}{2}\phi_e^2 \left[(\theta-1)\kappa_1 A_x + B_1^{\$,n-1}\right]^2 \\
&\quad + \frac{1}{2}(B_4^{\$,n-1}\phi_{\pi g})^2 - (\gamma + \phi_{\pi g}) B_4^{\$,n-1}\phi_{\pi g} \\
B_3^{\$,n} &= B_3^{\$,n-1}\rho_q + (\theta-1)A_q(\kappa_1\rho_q - 1) + \frac{1}{2}\left[(\theta-1)\kappa_1 A_\sigma + B_2^{\$,n-1} + \phi_{\phi\sigma}\left(B^{\$,n-1} - 1\right)\right]^2 \\
&\quad + \frac{1}{2}\left[(\theta-1)\kappa_1 A_q + B_3^{\$,n-1}\right]^2 \phi_q^2 \\
B_4^{\$,n} &= \phi_\pi \left(B^{\$,n-1} - 1\right).
\end{aligned}
\tag{73}
$$

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Table 1: Model Calibration

This table presents the calibrated parameters used in previous studies and in our paper. The column "BY" refers to the choice of parameters in Bansal and Yaron (2004), the column "BTZ" – to that in Bollerslev, Tauchen, and Zhou (2009), and the column "Our choice" refers to our choice of calibration parameters.

Type	Parameters	BY	BTZ	Our choice
Panel A: Real Economy				
	δ	0.997	0.997	0.997
Preferences	γ	10	10	8
	ψ	1.5	1.5	1.5
	μ_g	0.0015	0.0015	0.0015
	ρ_x	0.979	0	0.979
Endowment	ϕ_e	0.044	0	0.044
	a_σ	0.134×10^{-5}	0.134×10^{-5}	0.134×10^{-5}
	ρ_σ	0.978	0.978	0.978
	a_q	n/a	2×10^{-7}	2×10^{-7}
Uncertainty	ρ_q	n/a	0.8	0.8
	ϕ_q	n/a	0.001	0.001
Panel B: Inflation dynamics				
Constant	a_π	n/a	n/a	8×10^{-5}
Persistence	ρ_π	n/a	n/a	0.95
Autonomous	ϕ_π	n/a	n/a	0.0013
Consumption	$\phi_{\pi g}$	n/a	n/a	-0.0385
Uncertainty	$\phi_{\pi\sigma}$	n/a	n/a	28.50
Panel C: Campbell-Shiller constants				
	κ_0	0.3251	0.3251	0.3251
	κ_1	0.9	0.9	0.9

Table 2: Summary Statistics

This table presents summary statistics for the data used in the study. Panel A presents a summary statistics for the Treasury 1-year and 1-month excess bond returns for maturities 2 to 5 years; Panel B reports the macro-variables and variance risk premium-related series statistics. In Panel B FS_j, $j = 1, \ldots, 4$ refers to the Fama-Bliss j-year forward spreads, CP is the Cochrane-Piazzesi factor, IVAR is the squared implied volatility of S&P500 index, EVAR is the projected value of the realized market variance based on the HAR-RV model outlined in equation (34), VRP is the variance risk premium. Sample period is January 1990 to December 2012, frequency is monthly. Excess bond returns, forward spreads, and Cochrane-Piazzesi factor are computed using Fama-Bliss Treasury Bond data set from CRSP.

Panel A: Summary Statistics of Treasury Bond Returns

	1-year excess returns				1-month excess returns			
	2yr	3yr	4yr	5yr	2yr	3yr	4yr	5yr
Mean	0.92	1.78	2.56	3.06	1.09	1.62	2.07	2.27
Max	3.64	7.31	10.29	12.54	4.63	5.15	5.95	9.15
Min	-2.37	-5.24	-6.88	-8.37	-1.06	-1.37	-1.67	-2.92
Std. Dev.	1.33	2.53	3.53	4.38	0.88	1.21	1.58	1.92
Skewness	-0.08	-0.25	-0.35	-0.46	0.33	0.27	0.24	0.30
Kurtosis	2.23	2.51	2.60	2.72	3.45	2.73	2.43	2.77
AR(1) coeff	0.95	0.94	0.93	0.92	0.78	0.70	0.63	0.58

Panel B: Summary Statistics for Macro Factors and Variance Risk Premium

	FS_2	FS_3	FS_4	FS_5	CP	$IVAR$	$EVAR$	VRP
Mean	0.56	1.06	1.48	1.65	2.08	39.79	21.32	18.47
Max	2.04	3.25	3.84	4.32	4.66	298.90	282.60	206.97
Min	-0.77	-0.61	-0.60	-0.82	-0.40	9.05	3.92	-40.85
Std. Dev.	0.59	0.90	1.18	1.39	1.09	35.61	25.13	21.90
Skewness	0.13	0.30	0.19	0.14	0.46	3.32	5.57	3.74
Kurtosis	2.71	2.20	1.87	1.72	2.82	19.00	49.48	26.79
AR(1) coeff	0.92	0.96	0.96	0.97	0.91	0.80	0.75	0.28

Table 3: Bond Return Predictability with Equity VRP: 1-month Holding Period

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=1}^{2} \beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury bonds, $h = 1$ month and $\tau = 2, \ldots, 5$ years. VRP_t is the expected market variance risk premium, $F_{t,j}$, $j = 1, 2$ is the Cochrane-Piazzesi and the forward spread factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. The sample spans the period from January 1990 to December 2012, frequency of the data is monthly. Treasury excess returns are computed using Fama-Bliss data set.

	2yr				3yr			
Const	0.011	0.004	0.000	0.004	0.016	0.004	-0.001	-0.001
	(9.26)	(6.28)	(0.26)	(3.29)	(9.90)	(4.67)	(-0.93)	(-0.57)
VRP	-0.232	0.185	-0.155	0.173	0.242	0.279	0.356	0.352
	(-0.86)	(0.87)	(-0.90)	(0.92)	(0.67)	(1.15)	(1.73)	(1.71)
FS		1.225		1.180		1.083		0.076
		(21.23)		(5.85)		(20.32)		(0.45)
CP			0.623	0.026			0.926	0.865
			(16.73)	(0.24)			(24.33)	(6.38)
Adj. R2	-0.03	67.81	60.00	67.71	-0.17	65.41	69.49	69.40

	4yr				5yr			
Const	0.020	0.004	-0.001	0.000	0.021	0.004	-0.003	-0.000
	(9.86)	(3.71)	(-0.75)	(0.16)	(8.81)	(2.68)	(-1.47)	(-0.27)
VRP	0.494	0.480	0.637	0.589	1.175	0.862	1.336	1.089
	(0.97)	(1.46)	(2.31)	(2.10)	(1.47)	(1.67)	(2.47)	(2.20)
FS		1.037		0.344		1.016		0.552
		(19.10)		(2.28)		(17.43)		(4.59)
CP			1.161	0.812			1.300	0.679
			(21.49)	(4.91)			(16.92)	(4.57)
Adj. R2	0.11	60.94	64.35	65.15	1.44	55.90	55.59	59.22

Table 4: Bond Return Predictability with Equity VRP: 3-month Holding Period

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=1}^{2}\beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury bonds, $h = 3$ months and $\tau = 2, \ldots, 5$ years. VRP_t is the expected market variance risk premium, $F_{t,j}$, $j = 1, 2$ is the Cochrane-Piazzesi and the forward spread factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. The sample spans the period from January 1990 to December 2012, frequency of the data is monthly. Treasury excess returns are computed using Fama-Bliss data set.

	2yr				3yr			
Const	0.009	0.002	-0.000	0.001	0.015	0.003	-0.001	-0.001
	(8.18)	(2.94)	(-0.37)	(1.21)	(7.79)	(1.67)	(-0.52)	(-0.80)
VRP	0.071	0.455	0.055	0.315	0.457	0.501	0.432	0.421
	(0.25)	(2.21)	(0.34)	(1.74)	(1.05)	(1.66)	(1.68)	(1.62)
FS		1.115		0.728		1.094		-0.179
		(12.24)		(4.19)		(9.12)		(-0.43)
CP			0.572	0.225			0.910	1.052
			(12.71)	(2.84)			(10.05)	(3.31)
Adj. R2	-0.34	58.76	55.81	60.16	0.08	42.89	45.58	45.43
	4yr				5yr			
Const	0.019	0.004	-0.001	-0.000	0.021	0.004	-0.002	-0.001
	(7.44)	(1.45)	(-0.30)	(-0.16)	(6.55)	(1.34)	(-0.56)	(-0.29)
VRP	0.685	0.678	0.653	0.657	1.209	0.896	1.173	1.054
	(1.07)	(1.51)	(1.53)	(1.55)	(1.17)	(1.20)	(1.49)	(1.42)
FS		1.052		0.187		1.032		0.432
		(8.15)		(0.66)		(7.50)		(1.83)
CP			1.176	0.992			1.339	0.875
			(8.56)	(3.10)			(7.55)	(2.82)
Adj. R2	0.13	34.47	38.23	38.16	0.61	29.23	31.40	32.50

Table 5: Bond Return Predictability with Equity VRP: 1-year Holding Period

This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=1}^{2}\beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury bonds, $h = 1$ year and $\tau = 2, \ldots, 5$ years. VRP_t is the expected market variance risk premium, $F_{t,j}$, $j = 1, 2$ is the Cochrane-Piazzesi and the forward spread factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. The sample spans the period from January 1990 to December 2012, frequency of the data is monthly. Treasury returns are computed using Fama-Bliss data set.

		2yr				3yr		
Const	0.009	0.007	-0.002	-0.002	0.016	0.011	-0.002	-0.001
	(4.56)	(2.66)	(-0.70)	(-0.64)	(4.60)	(2.31)	(-0.34)	(-0.28)
VRP	0.342	0.444	0.291	0.185	0.848	0.878	0.761	0.740
	(0.86)	(1.20)	(0.75)	(0.46)	(1.10)	(1.19)	(1.00)	(0.97)
FS		0.278		-0.272		0.439		-0.180
		(0.96)		(-0.81)		(1.31)		(-0.46)
CP			0.510	0.586			0.873	0.954
			(5.01)	(4.82)			(4.69)	(4.38)
Adj. R2	-0.05	1.11	17.09	17.87	0.18	2.32	14.10	14.07
		4yr				5yr		
Const	0.024	0.014	-0.002	-0.002	0.028	0.017	-0.000	-0.002
	(4.77)	(2.09)	(-0.26)	(-0.27)	(4.65)	(2.27)	(-0.04)	(-0.23)
VRP	1.100	1.112	0.977	0.980	1.365	1.166	1.228	1.151
	(0.98)	(1.07)	(0.91)	(0.93)	(0.93)	(0.85)	(0.91)	(0.86)
FS		0.627		0.041		0.671		0.311
		(1.86)		(0.11)		(2.00)		(0.94)
CP			1.228	1.204			1.375	1.219
			(4.77)	(4.31)			(4.29)	(4.10)
Adj. R2	0.10	4.31	14.21	13.90	0.10	4.50	11.54	12.08

Table 6: Bond Return Predictability with Swaptions VRP: 1-month Holding Period

This table presents regression results for the following regression: $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)VRP_t + \sum_{j=1}^{2}\beta^{(\tau)}_j(h)F_{t,j} + \epsilon^{(\tau)}_{t+h}$, where $rx^{(\tau)}_{t+h}$ are excess returns on Treasury bonds, $h = 1$ month and $\tau = 2, \ldots, 5$ years. VRP_t is the expected variance risk premium derived from swaptions market, $F_{t,j}$, $j = 1, 2$ is the Cochrane-Piazzesi and the forward spread factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. The sample is from February 2005 to December 2012, monthly frequency. Treasury returns are computed using Fama-Bliss data set.

Panel A: maturity = 2 years						
Const	-0.023	-0.019	-0.028	-0.026	-0.031	-0.031
	(-4.01)	(-6.18)	(-3.66)	(-8.53)	(-4.44)	(-9.09)
VRP	0.005	NaN	NaN	0.003	0.004	0.003
	(3.00)	(NaN)	(NaN)	(3.98)	(2.52)	(3.10)
FS	NaN	2.762	NaN	2.405	NaN	2.212
	(NaN)	(6.10)	(NaN)	(7.18)	(NaN)	(7.15)
CP	NaN	NaN	0.670	NaN	0.480	0.311
	(NaN)	(NaN)	(3.14)	(NaN)	(2.61)	(2.87)
Adj. R2	28.86	53.88	27.73	67.71	40.82	72.51
Panel B: maturity = 3 years						
Const	-0.022	-0.025	-0.027	-0.030	-0.031	-0.034
	(-3.70)	(-6.38)	(-3.61)	(-8.67)	(-4.31)	(-9.31)
VRP	0.005	NaN	NaN	0.003	0.003	0.003
	(2.91)	(NaN)	(NaN)	(2.86)	(2.39)	(2.38)
FS	NaN	1.762	NaN	1.547	NaN	1.412
	(NaN)	(6.51)	(NaN)	(7.65)	(NaN)	(7.71)
CP	NaN	NaN	0.690	NaN	0.507	0.267
	(NaN)	(NaN)	(3.25)	(NaN)	(2.76)	(2.67)
Adj. R2	25.38	55.69	26.72	65.46	37.52	68.37
Panel C: maturity = 4 years						
Const	-0.021	-0.028	-0.027	-0.034	-0.030	-0.038
	(-3.36)	(-6.73)	(-3.52)	(-10.54)	(-4.12)	(-11.95)
VRP	0.005	NaN	NaN	0.003	0.003	0.003
	(2.68)	(NaN)	(NaN)	(4.37)	(2.15)	(3.33)
FS	NaN	1.309	NaN	1.184	NaN	1.093
	(NaN)	(6.85)	(NaN)	(8.13)	(NaN)	(8.03)
CP	NaN	NaN	0.705	NaN	0.526	0.270
	(NaN)	(NaN)	(3.30)	(NaN)	(2.82)	(2.77)
Adj. R2	21.04	51.53	23.42	61.87	31.94	64.30
Panel D: maturity = 5 years						
	(-3.33)	(-8.31)	(-3.46)	(-9.90)	(-4.13)	(-10.36)
VRP	0.005	NaN	NaN	0.002	0.004	0.002
	(3.04)	(NaN)	(NaN)	(1.74)	(2.51)	(1.44)
FS	NaN	1.168	NaN	1.051	NaN	0.977
	(NaN)	(8.06)	(NaN)	(7.32)	(NaN)	(6.98)
CP	NaN	NaN	0.738	NaN	0.542	0.232
	(NaN)	(NaN)	(3.38)	(NaN)	(2.81)	(2.04)
Adj. R2	20.58	57.09	21.66	59.46	30.29	60.72

Table 7: Bond Return Predictability with Swaptions VRP: 3-month Holding Period

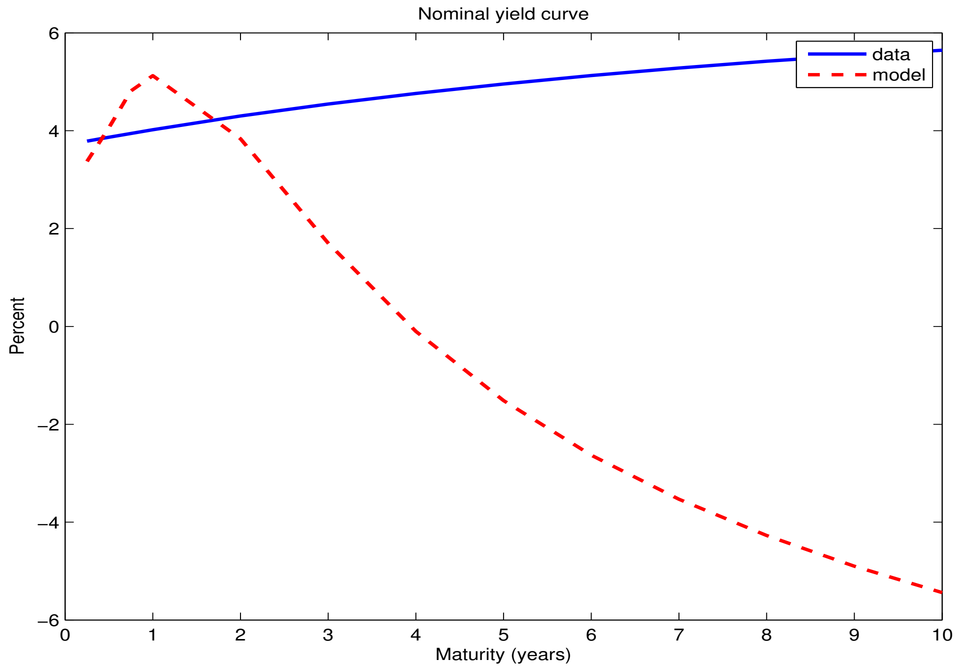
This table presents regression results for the following regression: $rx_{t+h}^{(\tau)} = \beta_0^{(\tau)} + \beta_1^{(\tau)}(h)VRP_t + \sum_{j=1}^{2}\beta_j^{(\tau)}(h)F_{t,j} + \epsilon_{t+h}^{(\tau)}$, where $rx_{t+h}^{(\tau)}$ are excess returns on Treasury bonds, $h = 3$ months and $\tau = 2, \ldots, 5$ years. VRP_t is the expected variance risk premium derived from swaptions market, $F_{t,j}$, $j = 1, 2$ is the Cochrane-Piazzesi and the forward spread factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. The sample is from February 2005 to December 2012, monthly frequency. Treasury excess returns are computed using Fama-Bliss data set.

Panel A: maturity = 2 years						
Const	-0.015	-0.013	-0.022	-0.018	-0.024	-0.023
	(-2.95)	(-4.48)	(-3.11)	(-5.41)	(-3.54)	(-6.64)
VRP	0.004	NaN	NaN	0.002	0.002	0.002
	(2.54)	(NaN)	(NaN)	(3.34)	(1.99)	(2.43)
FS	NaN	2.353	NaN	2.113	NaN	1.925
	(NaN)	(5.40)	(NaN)	(5.93)	(NaN)	(6.08)
CP	NaN	NaN	0.590	NaN	0.463	0.318
	(NaN)	(NaN)	(2.92)	(NaN)	(2.57)	(3.10)
Adj. R2	19.16	49.98	26.81	57.39	32.91	63.59
Panel B: maturity = 3 years						
Const	-0.012	-0.018	-0.019	-0.020	-0.021	-0.024
	(-1.95)	(-4.20)	(-2.50)	(-4.48)	(-2.74)	(-5.49)
VRP	0.003	NaN	NaN	0.001	0.002	0.001
	(2.16)	(NaN)	(NaN)	(1.82)	(1.52)	(1.14)
FS	NaN	1.633	NaN	1.530	NaN	1.402
	(NaN)	(5.56)	(NaN)	(5.81)	(NaN)	(5.96)
CP	NaN	NaN	0.600	NaN	0.500	0.269
	(NaN)	(NaN)	(2.70)	(NaN)	(2.42)	(2.60)
Adj. R2	10.24	46.73	19.11	48.28	21.16	50.87
Panel C: maturity = 4 years						
Const	-0.008	-0.020	-0.016	-0.022	-0.017	-0.025
	(-1.06)	(-4.21)	(-1.92)	(-4.21)	(-1.99)	(-5.13)
VRP	0.003	NaN	NaN	0.001	0.001	0.001
	(1.47)	(NaN)	(NaN)	(1.53)	(0.77)	(0.67)
FS	NaN	1.318	NaN	1.275	NaN	1.192
	(NaN)	(5.63)	(NaN)	(5.81)	(NaN)	(5.88)
CP	NaN	NaN	0.597	NaN	0.532	0.261
	(NaN)	(NaN)	(2.33)	(NaN)	(2.15)	(1.92)
Adj. R2	3.90	39.05	11.85	39.17	11.55	40.38
Panel D: maturity = 5 years						
	(-0.82)	(-3.45)	(-1.49)	(-3.30)	(-1.65)	(-3.78)
VRP	0.003	NaN	NaN	-0.000	0.002	-0.000
	(1.86)	(NaN)	(NaN)	(-0.13)	(1.24)	(-0.48)
FS	NaN	1.130	NaN	1.138	NaN	1.083
	(NaN)	(5.02)	(NaN)	(4.81)	(NaN)	(4.78)
CP	NaN	NaN	0.617	NaN	0.514	0.182
	(NaN)	(NaN)	(2.09)	(NaN)	(1.77)	(1.00)
Adj. R2	4.41	33.87	8.84	33.02	9.18	32.83

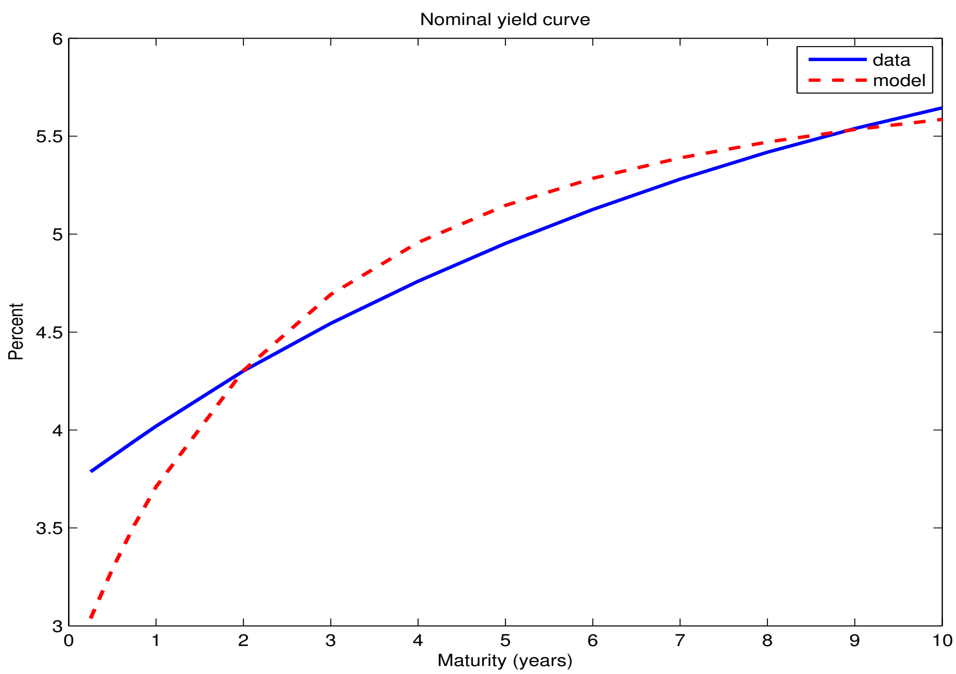
Table 8: Bond Return Predictability with Swaptions VRP: 1-year Holding Period

This table presents regression results for the following regression: $rx^{(\tau)}_{t+h} = \beta^{(\tau)}_0 + \beta^{(\tau)}_1(h)VRP_t + \sum_{j=1}^{2}\beta^{(\tau)}_j(h)F_{t,j} + \epsilon^{(\tau)}_{t+h}$, where $rx^{(\tau)}_{t+h}$ are excess returns on Treasury bonds, $h = 1$ year and $\tau = 2,\ldots,5$ years. VRP_t is the expected variance risk premium derived from swaptions market, $F_{t,j}$, $j = 1,2$ is the Cochrane-Piazzesi and the forward spread factors. t-statistics in parentheses are calculated using Newey and West (1987) standard errors. Adjusted R2 are given in percentage points. The sample is from February 2005 to December 2012, monthly frequency. Treasury excess returns are computed using Fama-Bliss data set.

Panel A: maturity = 2 years						
Const	0.012	0.011	0.010	0.012	0.011	0.011
	(4.68)	(4.69)	(3.16)	(4.32)	(3.19)	(3.21)
VRP	-0.001	NaN	NaN	-0.001	-0.001	-0.001
	(-1.41)	(NaN)	(NaN)	(-1.66)	(-2.00)	(-2.18)
FS	NaN	-0.123	NaN	-0.035	NaN	-0.079
	(NaN)	(-0.39)	(NaN)	(-0.12)	(NaN)	(-0.29)
CP	NaN	NaN	0.009	NaN	0.067	0.074
	(NaN)	(NaN)	(0.09)	(NaN)	(0.72)	(0.81)
Adj. R2	3.38	-0.79	-1.42	2.01	3.29	2.09
Panel B: maturity = 3 years						
Const	0.027	0.024	0.023	0.026	0.025	0.025
	(6.12)	(4.41)	(4.00)	(4.47)	(4.04)	(3.75)
VRP	-0.002	NaN	NaN	-0.002	-0.002	-0.002
	(-1.68)	(NaN)	(NaN)	(-2.36)	(-2.45)	(-2.79)
FS	NaN	-0.050	NaN	0.101	NaN	0.039
	(NaN)	(-0.14)	(NaN)	(0.32)	(NaN)	(0.13)
CP	NaN	NaN	0.008	NaN	0.128	0.121
	(NaN)	(NaN)	(0.05)	(NaN)	(0.78)	(0.79)
Adj. R2	5.41	-1.35	-1.44	4.38	5.61	4.25
Panel C: maturity = 4 years						
Const	0.042	0.032	0.035	0.037	0.038	0.035
	(6.90)	(3.68)	(4.23)	(3.99)	(4.30)	(3.55)
VRP	-0.003	NaN	NaN	-0.003	-0.003	-0.004
	(-1.89)	(NaN)	(NaN)	(-3.71)	(-3.10)	(-4.15)
FS	NaN	0.329	NaN	0.483	NaN	0.442
	(NaN)	(0.94)	(NaN)	(1.55)	(NaN)	(1.50)
CP	NaN	NaN	0.037	NaN	0.237	0.123
	(NaN)	(NaN)	(0.13)	(NaN)	(0.89)	(0.56)
Adj. R2	7.21	2.59	-1.38	14.13	8.45	13.49
Panel D: maturity = 5 years						
	(6.98)	(4.21)	(4.30)	(4.40)	(4.28)	(3.78)
VRP	-0.002	NaN	NaN	-0.004	-0.003	-0.004
	(-1.17)	(NaN)	(NaN)	(-3.76)	(-2.40)	(-4.18)
FS	NaN	0.305	NaN	0.568	NaN	0.499
	(NaN)	(0.89)	(NaN)	(1.76)	(NaN)	(1.78)
CP	NaN	NaN	0.196	NaN	0.383	0.212
	(NaN)	(NaN)	(0.51)	(NaN)	(1.06)	(0.72)
Adj. R2	1.86	2.20	-0.12	10.40	4.74	10.26



(a) No LRR component



(b) With LRR component

Figure 1: The model-implied yield curve without and with long-run risk component.

The figure plots the average zero-coupon nominal yield curve as observed in the data using the sample of January 1991 - December 2010 monthly data as the solid blue line and the model-implied yield curve without long-run risk component (Panel (a)) and with long-run risk component (Panel (b)) as a dashed red line.

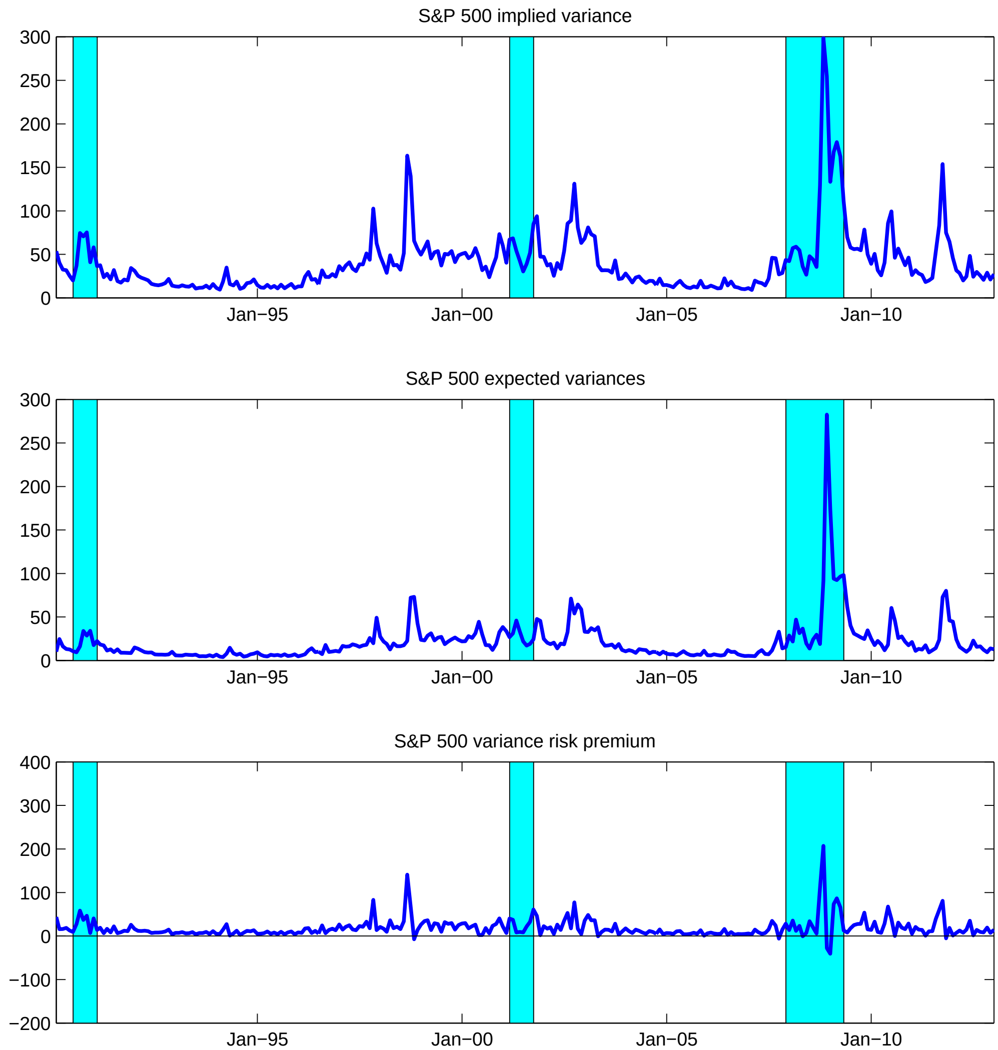


Figure 2: Equity variance risk premium

This figure plots the implied variance (top panel), the expected variance (middle panel), and their difference, the variance risk premium, (the bottom panel) for the S&P 500 index. Sample period is from January 1990 to December 2012. Blue shaded bars indicate NBER recessions.

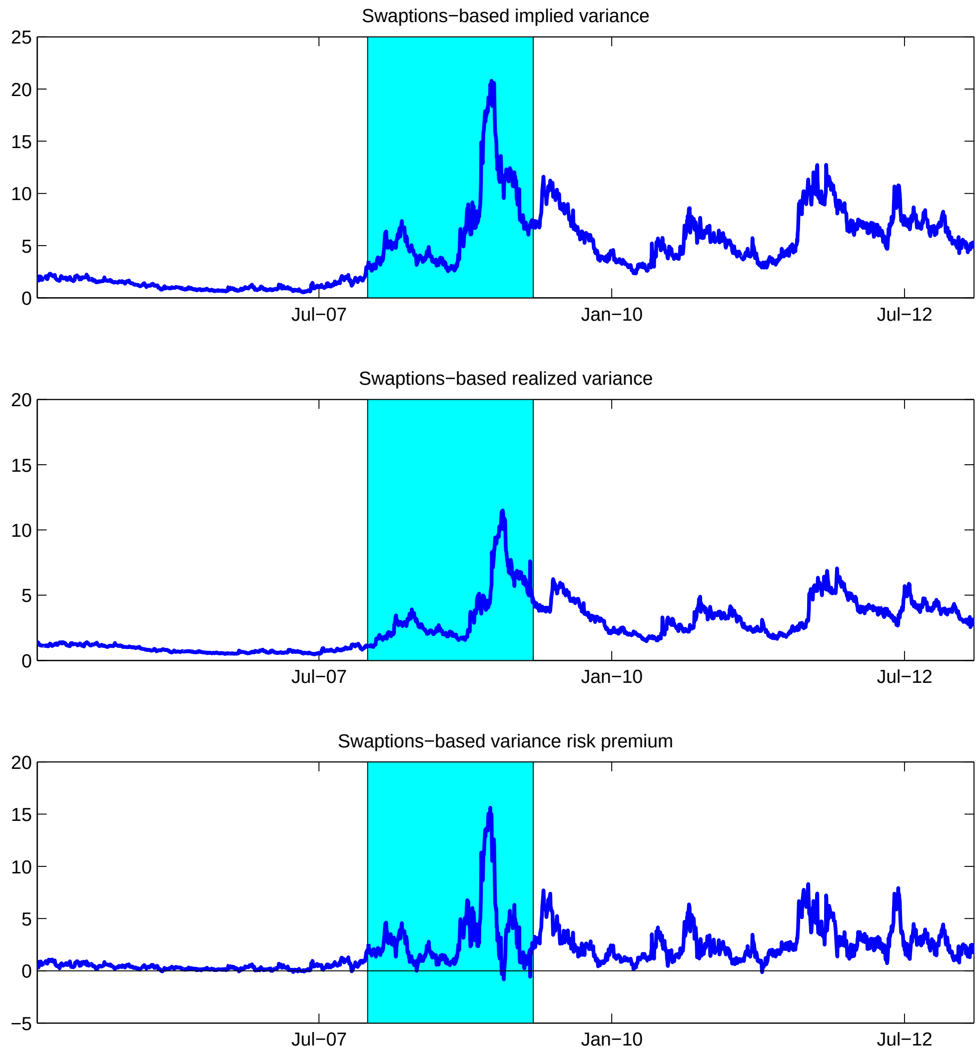


Figure 3: Swaptions-based variance risk premium

This figure plots the implied variance (top panel), the expected variance (middle panel), and their difference, the variance risk premium, (the bottom panel) derived from interest rate swaptions. The implied variance risk premium is corresponds to the one-month swaption on 10-year interest rate. Realized variance is derived from intraday 10-year interest rate swaps data. Sample period is from February 2005 to December 2012. Blue shaded bar indicates NBER recession.